# Package 'treenomial'

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Type Package

Title Comparison of Trees using a Tree Defining Polynomial

Version 1.1.4

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**Description** Provides functionality for creation and comparison of polynomials that uniquely describe trees as introduced in Liu (2019, <doi:10.48550/arXiv.1904.03332>). The core method converts rooted unlabeled phylo objects from 'ape' to the tree defining polynomials described with coefficient matrices. Additionally, a conversion for rooted binary trees with binary trait labels is also provided. Once the polynomials of trees are calculated there are functions to calculate distances, distance matrices and plot different distance trees from a target tree. Manipulation and conversion to the tree defining polynomials is implemented in C++ with 'Rcpp' and 'RcppArmadillo'. Furthermore, parallel programming with 'RcppThread' is used to improve performance converting to polynomials and calculating distances.

**Depends** R (>= 3.5.0)

License GPL (>= 2)

URL https://github.com/gouldmatt/treenomial

**Encoding** UTF-8

**Imports** Rcpp (>= 1.0.1), ape, methods

LinkingTo Rcpp, RcppArmadillo, RcppThread (>= 2.1.3)

RoxygenNote 7.2.0

Suggests knitr, rmarkdown, testthat

NeedsCompilation yes

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**Repository** CRAN

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## alignPoly

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```
alignPoly
```

Align various types of coefficient matrices

## Description

Align various types of coefficient matrices

## Usage

alignPoly(coefficientMatrices)

#### Arguments

coefficientMatrices a list of coefficient matrices of various sizes

## Details

Alignment depends on the type of coefficient matrix:

- "real" the smaller matrices columns are prepended with zero columns to align with the max number of columns and the rows are appended with zero rows to match the max number of rows
- "yEvaluated" the smaller vectors are appended with zeroes to match the max length vector
- "tipLabel" the smaller matrices are appended with zeroes to match the max number of rows and columns

#### Value

the aligned list of coefficient matrices

## allTrees

## Examples

```
library(treenomial)
library(ape)
differentSizeTrees <- c(rtree(2), rmtree(10,10))
coeffs <- treeToPoly(differentSizeTrees, numThreads = 0)
alignedCoeffs <- alignPoly(coeffs)</pre>
```

allTrees

```
Calculate all full unordered m-ary trees up to n tips
```

#### Description

Return normal coefficient matrices, substituted y coefficient vectors, or phylo objects for all possible unordered full m-ary trees up to n tips. For binary trees (m = 2), the number of trees at each number of tips follows the Wedderburn-Etherington numbers.

#### Usage

```
allTrees(n, m = 2, type = c("default", "yEvaluated", "phylo"), y)
```

## Arguments

n	max number of tips
m	max number of children for each node
type	one of:
	"real" tree distiguishing polynomials in two variables x (columns) and y (rows)
	"yEvaluated" tree distiguishing polynomials with y evaluated at a specified argument
	"phylo" phylo objects
У	the y value to evaluate the polynomial at when type is "yEvaluated", ignored otherwise

## Value

list of lists containing all the trees in type format for each number of tips

#### Note

only m = 2 is currently supported

#### Examples

```
library(treenomial)
library(ape)
# generate coefficient matrices describing the polynomials of all possible
# unordered full binary trees up to 10 tips
allBinTenRealCoeff <- allTrees(10, type = "phylo")</pre>
# number of trees at each number of tips follows Wedderburn-Etherington numbers
lengths(allBinTenRealCoeff)
# phylo type example, plot all 6 tip unordered full binary trees
# backup par options
oldpar <- par(no.readonly =TRUE)</pre>
allBinSixPhylo <- allTrees(6, type = "phylo")[[6]]</pre>
par(mfrow=c(1,6))
plots <- lapply(allBinSixPhylo, function(t){</pre>
  plot.phylo(ladderize(t), direction = "downwards", show.tip.label = FALSE)
})
# restore par options
par(oldpar)
```

plotExtremeTrees	Plot the min/max distance	trees from a target tree
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## Description

Plot the min/max distance trees from a target tree

#### Usage

```
plotExtremeTrees(
  target,
  trees,
  n,
  comparison = "min",
  method = c("fraction", "logDiff", "wLogDiff", "pa", "ap"),
  type = c("default", "yEvaluated", "tipLabel"),
  y,
  numThreads = -1
)
```

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## plotExtremeTrees

#### Arguments

target	the phylo object of the tree to calculate the distances to
trees	a list of phylo objects to compare with the target
n	the number of trees to find and plot
comparison	whether to find the "min" or the "max" distance trees from the target
method	method to use when calculating coefficient distances:
	<b>"fraction"</b> for two coefficient matrices A and B returns sum(abs(A-B)/(A+B)), excluding elements where both A and B are zero
	"logDiff" for two coefficient matrices A and B returns sum(log(1+abs(A-B))
	"wLogDiff" performs the "logDiff" method with weights on the rows
	<b>"pa"</b> total pairs where the coefficient is present in one matrix and absent in the other (presence-absence)
	"ap" opposite comparison of pa (absence-presence)
type	one of:
	"real" tree distinguishing polynomials in two variables x (columns) and y (rows)
	"yEvaluated" tree distinguishing polynomials with y evaluated at a specified argument
	<b>"tipLabel"</b> complex coefficient polynomial that utilize binary trait tip labels on the phylo objects
У	the y value to evaluate the polynomial at when type is "yEvaluated", ignored otherwise
numThreads	number of threads to be used, the default $(-1)$ will use the number of cores in the machine and numThreads = 0 will only use the main thread

## Value

a list of lists containing the n min/max distance trees and their distances to target

## Note

- the substituted y coefficient vector only supports the "logDiff" method and the "fraction" method
- "pa" and "ap" force symmetry in the output distance matrix

```
library(treenomial)
library(ape)
trees <- c(rmtree(1000, 50), rmtree(10, 9))
target <- rtree(50)
minTrees <- plotExtremeTrees(target, trees, 2, comparison = "min", numThreads = 0)</pre>
```

polyDist

## Description

Calculates the distance between two coefficient matrices or a coefficient matrix and a list of coefficient matrices.

## Usage

```
polyDist(
    x,
    Y,
    method = c("fraction", "logDiff", "wLogDiff", "pa", "ap"),
    numThreads = -1
)
```

## Arguments

Х	single coefficient matrix to find distances to
Υ	a list of coefficient matrices
method	method to use when calculating coefficient distances:
	<b>"fraction"</b> for two coefficient matrices A and B returns sum(abs(A-B)/(A+B)), excluding elements where both A and B are zero
	"logDiff" for two coefficient matrices A and B returns sum(log(1+abs(A-B))
	"wLogDiff" performs the "logDiff" method with weights on the rows
	<b>"pa"</b> total pairs where the coefficient is present in one matrix and absent in the other (presence-absence)
	"ap" opposite comparison of pa (absence-presence)
numThreads	number of threads to be used, the default $(-1)$ will use the number of cores in the machine and numThreads = 0 will only use the main thread

## Value

vector of distances

## Note

- the substituted y coefficient vector only supports the "logDiff" method and the "fraction" method
- "pa" and "ap" force symmetry in the output distance matrix

## polyToDistMat

## Examples

```
library(treenomial)
library(ape)
# distance between coefficient matrices of one 10 tip tree
# and 100 trees with 30 tips using
# create the coefficient matrices
tenTipTree <- rtree(10)
tenTipTreeCoeff <- treeToPoly(tenTipTree, numThreads = 0)
thirtyTipList <- rmtree(100, 30)
thirtyTipCoeffs <- treeToPoly(thirtyTipList, numThreads = 0)
# find the distance
polyDist(tenTipTreeCoeff, thirtyTipCoeffs, numThreads = 0)</pre>
```

polyToDistMat

Calculates the distance matrix from a list coefficient matrices

## Description

Calculates the distance matrix from a list coefficient matrices

#### Usage

```
polyToDistMat(
  coefficientMatrices,
  method = c("fraction", "logDiff", "wLogDiff", "pa", "ap"),
  numThreads = -1
)
```

coefficientMat	rices
	list of coefficient matrices
method	method to use when calculating coefficient distances:
	<b>"fraction"</b> for two coefficient matrices A and B returns sum(abs(A-B)/(A+B)), excluding elements where both A and B are zero
	"logDiff" for two coefficient matrices A and B returns sum(log(1+abs(A-B))
	"wLogDiff" performs the "logDiff" method with weights on the rows
	<b>"pa"</b> total pairs where the coefficient is present in one matrix and absent in the other (presence-absence)
	"ap" opposite comparison of pa (absence-presence)
numThreads	number of threads to be used, the default $(-1)$ will use the number of cores in the machine and numThreads = 0 will only use the main thread

distance matrix calculated from argument coefficient matrices

#### Note

- the substituted y coefficient vector only supports the "logDiff" method and the "fraction" method
- "pa" and "ap" force symmetry in the output distance matrix

#### Examples

```
library(treenomial)
library(ape)
# coefficient matrices for ten trees of 20 tips
coeffs <- treeToPoly(rmtree(10, 20), numThreads = 0)
# distance matrix from the list of coefficient matrices
d <- polyToDistMat(coeffs, method = "logDiff", numThreads = 0)
# using the absence-presence method
d <- polyToDistMat(coeffs, method = "ap", numThreads = 0)</pre>
```

treeDist

Calculates the distance between trees

#### Description

Calculates the distance between two trees or a tree and a list of trees.

#### Usage

```
treeDist(
    x,
    Y,
    type = c("default", "yEvaluated", "tipLabel"),
    method = c("fraction", "logDiff", "wLogDiff", "pa", "ap"),
    y,
    numThreads = -1
)
```

х	single phylo object
Y	a list of phylo objects
type	one of:
	"real" tree distinguishing polynomials in two variables x (columns) and y (rows)

#### treeDist

	"yEvaluated" tree distinguishing polynomials with y evaluated at a specified argument
	<b>"tipLabel"</b> complex coefficient polynomial that utilize binary trait tip labels on the phylo objects
method	method to use when calculating coefficient distances:
	<b>"fraction"</b> for two coefficient matrices A and B returns sum(abs(A-B)/(A+B)), excluding elements where both A and B are zero
	<b>"logDiff"</b> for two coefficient matrices A and B returns sum(log(1+abs(A-B))
	"wLogDiff" performs the "logDiff" method with weights on the rows
	<b>"pa"</b> total pairs where the coefficient is present in one matrix and absent in the other (presence-absence)
	"ap" opposite comparison of pa (absence-presence)
У	the y value to evaluate the polynomial at when type is "yEvaluated", ignored otherwise
numThreads	number of threads to be used, the default $(-1)$ will use the number of cores in the machine and numThreads = 0 will only use the main thread

## Value

vector of distances

### Note

- the substituted y coefficient vector only supports the "logDiff" method and the "fraction" method
- "pa" and "ap" force symmetry in the output distance matrix

```
library(treenomial)
library(ape)
# distance between one 10 tip tree and 100 trees with 30 tips
# generate the trees
tenTipTree <- rtree(10)
thirtyTipList <- rmtree(100, 30)
# find the distance
treeDist(tenTipTree, thirtyTipList, numThreads = 0)</pre>
```

treeJuliaSet

## Description

Finds the Julia Set for the y evaluated polynomial of a tree and plots in a square image.

## Usage

```
treeJuliaSet(
   tree,
   pixelLength = 700,
   center = 0,
   maxZ = 2,
   maxIter = 100,
   col = c("white", colorRampPalette(c("dodgerblue4", "lightblue"))(98), "black"),
   y
)
```

## Arguments

tree	phylo object
pixelLength	number of pixels on one side of the image
center	complex number giving the center of the image on the complex plane
maxZ	the max value for the real and imaginary axis
maxIter	maximum count for iterations
col	colours to be used for the image
У	the y value to evaluate the polynomial at

```
library(treenomial)
library(ape)
treeJuliaSet(stree(5,type = "right"), y = 1+1i)
```

treeToDistMat

## Description

Calculates the distance matrix from a list of phylo objects

## Usage

```
treeToDistMat(
   trees,
   method = c("fraction", "logDiff", "wLogDiff", "pa", "ap"),
   type = c("default", "yEvaluated", "tipLabel"),
   y,
   numThreads = -1
)
```

## Arguments

trees	a single phylo object or a list of phylo objects
method	method to use when calculating coefficient distances:
	<b>"fraction"</b> for two coefficient matrices A and B returns sum(abs(A-B)/(A+B)), excluding elements where both A and B are zero
	"logDiff" for two coefficient matrices A and B returns sum(log(1+abs(A-B))
	"wLogDiff" performs the "logDiff" method with weights on the rows
	<b>"pa"</b> total pairs where the coefficient is present in one matrix and absent in the other (presence-absence)
	"ap" opposite comparison of pa (absence-presence)
type	one of:
	"real" tree distinguishing polynomials in two variables x (columns) and y (rows)
	<b>"yEvaluated"</b> tree distinguishing polynomials with y evaluated at a specified argument
	<b>"tipLabel"</b> complex coefficient polynomial that utilize binary trait tip labels on the phylo objects
У	the y value to evaluate the polynomial at when type is "yEvaluated", ignored otherwise
numThreads	number of threads to be used, the default $(-1)$ will use the number of cores in the machine and numThreads = 0 will only use the main thread

#### Value

a distance matrix

- the substituted y coefficient vector only supports the "logDiff" method and the "fraction" method
- "pa" and "ap" force symmetry in the output distance matrix

## Examples

```
library(treenomial)
library(ape)
# distance matrix for 10 trees of 30 tips
treeToDistMat(rmtree(10, 30), method = "wLogDiff", numThreads = 0)
```

treeToPoly Convert trees to coefficient matrices

## Description

Converts rooted full binary trees to tree distinguishing polynomials described with coefficient matrices.

#### Usage

```
treeToPoly(
  trees,
  type = c("default", "yEvaluated", "tipLabel"),
  y,
  varLabels = FALSE,
  numThreads = -1
)
```

trees	a single phylo object or a list of phylo objects
type	one of:
	<ul> <li>"real" tree distinguishing polynomials in two variables x (columns) and y (rows)</li> <li>"yEvaluated" tree distinguishing polynomials with y evaluated at a specified argument</li> <li>"tipLabel" complex coefficient polynomial that utilize binary trait tip labels on the phylo objects</li> </ul>
У	the y value to evaluate the polynomial at when type is "yEvaluated", ignored otherwise
varLabels	boolean for whether to add row and column names corresponding to the variables in the polynomial
numThreads	number of threads to be used, the default $(-1)$ will use the number of cores in the machine and numThreads = 0 will only use the main thread

wedge

## Value

the resulting coefficient matrix or matrices of the form:

"real" a real matrix where the ith row, jth column represents the x^(j-1)\*y^(i-1) coefficient

"yEvaluated" a vector where the kth column represents the x^(k-1) coefficient

"**tipLabel**" given trees with two unique tip labels "a", "b" a complex matrix where the ith row, jth column represents the a^(i-1)\*b^(j-1) coefficient

## Examples

```
library(treenomial)
library(ape)
# generate a tree
tree <- rtree(n = 30, rooted = TRUE)
# a real coefficient matrix
treeToPoly(tree, varLabels = TRUE, numThreads = 0)
# complex coefficient vector for the tree
treeToPoly(tree, type = "yEvaluated", y = 1+1i, varLabels = TRUE, numThreads = 0)
# for a list of trees
treeToPoly(rmtree(4, 20), varLabels = TRUE, numThreads = 0)</pre>
```

WEURE
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Performs the wedge operation

## Description

Calculates the result from the wedge operation on two real coefficient matrices, two y evaluated polynomial coefficient vectors or two phylo objects.

#### Usage

```
wedge(A, B, type = c("default", "yEvaluated", "phylo"), y)
```

А, В	two real coefficient matrices, complex coefficient vectors or phylo objects
type	one of:
	"real" tree distinguishing polynomials in two variables x (columns) and y (rows)
	<b>"yEvaluated"</b> tree distinguishing polynomials with y evaluated at a specified argument
	<b>"tipLabel"</b> complex coefficient polynomial that utilize binary trait tip labels on the phylo objects
У	the y value to evaluate the polynomial at when type is "yEvaluated", ignored otherwise

wedge

## Value

the wedge result in the same form as the arguments

```
library(treenomial)
library(ape)
# wedge two real coefficient matrices
leaf <- matrix(c(0,1), nrow = 1, ncol = 2)
wedge(leaf, leaf)
# wedge two complex coefficient vectors
leaf <- as.complex(c(0,1))
wedge(leaf, leaf, "yEvaluated",5)</pre>
```

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