

Package ‘mcmcse’

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Title Monte Carlo Standard Errors for MCMC

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Description Provides tools for computing Monte Carlo standard errors (MCSE) in Markov chain Monte Carlo (MCMC) settings. MCSE computation for expectation and quantile estimators is supported as well as multivariate estimations. The package also provides functions for computing effective sample size and for plotting Monte Carlo estimates versus sample size.

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mcmcse-package	<i>Monte Carlo Standard Errors for MCMC</i>
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Description

Provides tools for computing Monte Carlo standard errors (MCSE) in Markov chain Monte Carlo (MCMC) settings. MCSE computation for expectation and quantile estimators is supported. The package also provides functions for computing effective sample size and for plotting Monte Carlo estimates versus sample size.

Details

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References

- Dai, N and Jones, G.L. (2017) Multivariate initial sequence estimators in Markov chain Monte Carlo, *Journal of Multivariate Analysis*, **159**, 184–199.
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- Doss, C. R., Flegal, J. M., Jones, G. L., and Neath, R. C. (2014). Markov chain Monte Carlo estimation of quantiles. *Electronic Journal of Statistics*, **8**, 2448–2478.
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- Heberle, J., and Sattarhoff, C. (2017). A fast algorithm for the computation of HAC covariance matrix estimators. *Econometrics*, **5**, 9.
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- Vats, D., Flegal, J. M., and Jones, G. L. Multivariate output analysis for Markov chain Monte Carlo, *Biometrika*, **106**, 321–337.
- Vats, D., Flegal, J. M., and Jones, G. L. (2018) Strong Consistency of multivariate spectral variance estimators for Markov chain Monte Carlo, *Bernoulli*, **24**, 1860–1909.

Examples

```
n <- 1e3
mu = c(2, 50)
sigma = matrix(c(1, 0.5, 0.5, 1), nrow = 2)
out = BVN_Gibbs(n, mu, sigma)

multiESS(out)
ess(out)
mcse.mat(out)

mcse.bm <- mcse.multi(x = out)
mcse.tuk <- mcse.multi(x = out, method = "tukey")
```

batchSize	<i>Batch size (truncation point) selection</i>
-----------	--

Description

Function returns the optimal batch size (or truncation point) for a given chain and method.

Usage

```
batchSize(x, method = c("bm", "obm", "bartlett", "tukey", "sub"), g = NULL, fast = TRUE)
```

Arguments

x	A matrix or data frame of Markov chain output. Number of rows is the Monte Carlo sample size.
method	Any of “bm”, “obm”, “bartlett”, “tukey”. “bm” represents batch means estimator, “obm” represents the overlapping batch means estimator, and “bartlett” and “tukey” represent the modified-Bartlett window and the Tukey-Hanning windows for the spectral variance estimators.
g	A function that represents features of interest. g is applied to each row of x and thus g should take a vector input only. If g is NULL, g is set to be identity, which is estimation of the mean of the target density.
fast	Boolean variable for fast estimation using a reasonable subset of the Markov chain.

Details

batchSize fits a stationary autoregressive process to the marginals of x, selecting the order of the process as the one with the maximum AIC among the models with coefficients greater than a threshold.

Value

A value of the optimal batch size (truncation point or bandwidth) is returned.

References

Liu, Y., Vats, D., and Flegal, J. M. (2021) Batch size selection for variance estimators in MCMC, *Methodology and Computing in Applied Probability*, to appear.

See Also

[mcse.multi](#) and [mcse](#), which calls on batchSize.

Examples

```
## Bivariate Normal with mean (mu1, mu2) and covariance sigma
n <- 1e3
mu <- c(2, 50)
sigma <- matrix(c(1, 0.5, 0.5, 1), nrow = 2)

out <- BVN_Gibbs(n, mu, sigma)

batchSize(out)
batchSize(out, method = "obm")
batchSize(out, method = "bartlett")
```

BVN_Gibbs

MCMC samples from a bivariate normal distribution

Description

Function returns Gibbs samples from a bivariate normal target density.

Usage

```
BVN_Gibbs(n, mu, sigma)
```

Arguments

n	Sample size of the Markov chain.
mu	A 2 dimensional vector. Mean of the target normal distribution.
sigma	2 x 2 symmetric positive definite matrix. The covariance matrix of the target normal distribution.

Value

An n x 2 matrix of the Gibbs samples.

Examples

```
n <- 1e3
mu <- c(2, 50)
sigma <- matrix(c(1, 0.5, 0.5, 1), nrow = 2)
out <- BVN_Gibbs(n, mu, sigma)
```

confRegion	<i>Confidence regions (ellipses) for Monte Carlo estimates</i>
------------	--

Description

Constructs confidence regions (ellipses) from the Markov chain output for the features of interest
Function uses the ellipse package.

Usage

```
confRegion(mcse.obj, which = c(1,2), level = .95)
```

Arguments

mcse.obj	The list returned by the mcse.multi or mcse.initseq command, or an mcmcse class object.
which	Integer vector of length 2 indicating the component for which to make the confidence ellipse. Chooses the first two by default.
level	confidence level for the ellipse.

Value

Returns a matrix of x and y coordinates for the ellipse. Use plot function on the matrix to plot the ellipse.

Examples

```
## Bivariate Normal with mean (mu1, mu2) and covariance sigma
n <- 1e3
mu <- c(2, 50)
sigma <- matrix(c(1, 0.5, 0.5, 1), nrow = 2)

out <- BVN_Gibbs(n, mu, sigma)

mcerror <- mcse.multi(out, blather = TRUE)

## Plotting the ellipse
plot(confRegion(mcerror), type = 'l')
```

ess	<i>Univariate effective sample size (ESS) as described in Gong and Flegal (2015).</i>
-----	---

Description

Estimate the effective sample size (ESS) of a Markov chain as described in Gong and Flegal (2015).

Usage

```
ess(x, g = NULL, ...)
```

Arguments

x	a matrix or data frame of Markov chain output. Number of rows is the Monte Carlo sample size.
g	a function that represents features of interest. g is applied to each row of x and thus g should take a vector input only. Ifcodeg is NULL, g is set to be identity, which is estimation of the mean of the target density.
...	arguments passed on to the mcse.mat function. For example method = “tukey” and size = “cuberoot” can be used.

Details

ESS is the size of an iid sample with the same variance as the current sample for estimating the expectation of g. ESS is given by

$$ESS = n \frac{\lambda^2}{\sigma^2}$$

where λ^2 is the sample variance and σ^2 is an estimate of the variance in the Markov chain central limit theorem. The denominator by default is a batch means estimator, but the default can be changed with the ‘method’ argument.

Value

The function returns the estimated effective sample size for each component of g.

References

Gong, L. and Flegal, J. M. (2015) A practical sequential stopping rule for high-dimensional Markov chain Monte Carlo, *Journal of Computational and Graphical Statistics*, **25**, 684—700.

See Also

[minESS](#), which calculates the minimum effective samples required for the problem. [multiESS](#), which calculates multivariate effective sample size using a Markov chain and a function g.

<code>estvssamp</code>	<i>Create a plot that shows how Monte Carlo estimates change with increasing sample size.</i>
------------------------	---

Description

Create a plot that shows how Monte Carlo estimates change with increasing sample size.

Usage

```
estvssamp(x, g = mean, main = "Estimates vs Sample Size", add = FALSE, ...)
```

Arguments

<code>x</code>	a sample vector.
<code>g</code>	a function such that $E(g(x))$ is the quantity of interest. The default is <code>g = mean</code> .
<code>main</code>	an overall title for the plot. The default is “Estimates vs Sample Size”.
<code>add</code>	logical. If TRUE, add to a current plot.
<code>...</code>	additional arguments to the plotting function.

Value

NULL

Examples

```
## Bivariate Normal with mean (mu1, mu2) and covariance sigma
n <- 1e3
mu <- c(2, 50)
sigma <- matrix(c(1, 0.5, 0.5, 1), nrow = 2)

out <- BVN_Gibbs(n, mu, sigma)

estvssamp(out[,1], main = expression(E(x[1])))
```

<code>is.mcmcse</code>	<i>Check if the class of the object is mcmcse</i>
------------------------	---

Description

Check if the class of the object is mcmcse

Usage

```
is.mcmcse(x)
```


Arguments

`x` The object that is checked to belong to the class `mcmcse`

Value

Boolean variable indicating if the input belongs to the class `mcmcse`

Examples

```
## Bivariate Normal with mean (mu1, mu2) and covariance sigma
n <- 1e3
mu <- c(2, 50)
sigma <- matrix(c(1, 0.5, 0.5, 1), nrow = 2)

out <- BVN_Gibbs(n, mu, sigma)
is.mcmcse(mcse.multi(out))
```

mcse

Compute Monte Carlo standard errors for expectations.

Description

Compute Monte Carlo standard errors for expectations.

Usage

```
mcse(x, size = NULL, g = NULL, r = 3, method = "bm", warn = FALSE)
```

Arguments

<code>x</code>	a vector of values from a Markov chain of length <code>n</code> .
<code>size</code>	represents the batch size in “bm” and the truncation point in “bartlett” and “tukey”. Default is <code>NULL</code> which implies that an optimal batch size is calculated using the <code>batchSize</code> function. Can take character values of “sqroot” and “cuberoor” or any numeric value between 1 and $n/2$. “sqroot” means size is $\lfloor n^{1/2} \rfloor$ and “cuberoor” means size is $\lfloor n^{1/3} \rfloor$.
<code>g</code>	a function such that $E(g(x))$ is the quantity of interest. The default is <code>NULL</code> , which causes the identity function to be used.
<code>r</code>	The lugsail parameters (r) that converts a lag window into its lugsail equivalent. Larger values of r will typically imply less underestimation of “cov”, but higher variability of the estimator. Default is $r = 3$ and $r = 1, 2$ are also good choices, but will likely underestimation of variance. $r > 5$ is not recommended.
<code>method</code>	any of “bm”, “obm”, “bartlett”, “tukey”. “bm” represents batch means estimator, “obm” represents overlapping batch means estimator with, “bartlett” and “tukey” represents the modified-Bartlett window and the Tukey-Hanning windows for spectral variance estimators.

warn a logical value indicating whether the function should issue a warning if the sample size is too small (less than 1,000).

Value

mcse returns a list with three elements:

est an estimate of $E(g(x))$.
se the Monte Carlo standard error.
nsim The number of samples in the input Markov chain.

References

- Flegal, J. M. (2012) Applicability of subsampling bootstrap methods in Markov chain Monte Carlo. In Wozniakowski, H. and Plaskota, L., editors, *Monte Carlo and Quasi-Monte Carlo Methods 2010*, pp. 363-372. Springer-Verlag.
- Flegal, J. M. and Jones, G. L. (2010) Batch means and spectral variance estimators in Markov chain Monte Carlo. *The Annals of Statistics*, **38**, 1034–1070.
- Flegal, J. M. and Jones, G. L. (2011) Implementing Markov chain Monte Carlo: Estimating with confidence. In Brooks, S., Gelman, A., Jones, G. L., and Meng, X., editors, *Handbook of Markov Chain Monte Carlo*, pages 175–197. Chapman & Hall/CRC Press.
- Doss, C. R., Flegal, J. M., Jones, G. L., and Neath, R. C. (2014). Markov chain Monte Carlo estimation of quantiles. *Electronic Journal of Statistics*, **8**, 2448-2478. Jones, G. L., Haran, M., Caffo, B. S. and Neath, R. (2006) Fixed-width output analysis for Markov chain Monte Carlo. *Journal of the American Statistical Association*, **101**, 1537–154.

See Also

[mcse.mat](#), which applies mcse to each column of a matrix or data frame.
[mcse.multi](#), for a multivariate estimate of the Monte Carlo standard error.
[mcse.q](#) and [mcse.q.mat](#), which compute standard errors for quantiles.

Examples

```
## Bivariate Normal with mean (mu1, mu2) and covariance sigma
n <- 1e3
mu = c(2, 50)
sigma = matrix(c(1, 0.5, 0.5, 1), nrow = 2)
out = BVN_Gibbs(n, mu, sigma)
x = out[,1]
mcse(x)
mcse.q(x, 0.1)
mcse.q(x, 0.9)

# Estimate the mean, 0.1 quantile, and 0.9 quantile with MCSEs using overlapping batch means.

mcse(x, method = "obm")
mcse.q(x, 0.1, method = "obm")
mcse.q(x, 0.9, method = "obm")
```

```
# Estimate E(x^2) with MCSE using spectral methods.

g = function(x) { x^2 }
mcse(x, g = g, method = "tukey")
```

mcse.initseq	<i>Multivariate Monte Carlo standard errors for expectations with the initial sequence method of Dai and Jones (2017)</i>
--------------	---

Description

Function returns the estimate of the covariance matrix in the Markov Chain central limit theorem using initial sequence method. This method is designed to give an asymptotically conservative estimate of the Monte Carlo standard error.

Usage

```
mcse.initseq(x, g = NULL, adjust = FALSE, blather = FALSE)
```

Arguments

x	A matrix or data frame of Markov chain output. Number of rows is the Monte Carlo sample size.
g	A function that represents features of interest. g is applied to each row of x and thus g should take a vector input only. If g is NULL, g is set to be identity, which is estimation of the mean of the target density.
adjust	Logical; if TRUE, an adjustment is made to increase slightly the eigenvalues of the initial sequence estimator. The default is FALSE.
blather	if TRUE, outputs under the hood information about the function.

Value

A list is returned with the following components,

cov	a covariance matrix estimate using initial sequence method.
cov.adj	a covariance matrix estimate using adjusted initial sequence method if the input adjust=TRUE.
eigen_values	eigen values of the estimate cov.
method	method used to calculate matrix cov.
est	estimate of g(x).
nsim	number of rows of the input x. Only if blather = TRUE.
Adjustment_Used	logical of whether an adjustment was made to the initial sequence estimator. Only if blather = TRUE.

References

Dai, N and Jones, G.L. (2017) Multivariate initial sequence estimators in Markov chain Monte Carlo, *Journal of Multivariate Analysis*, **159**, 184-199.

See Also

`mcse`, which acts on a vector. `mcse.mat`, which applies `mcse` to each column of a matrix or data frame. `mcse.q` and `mcse.q.mat`, which compute standard errors for quantiles. `mcse.multi`, which estimates the covariance matrix in the Markov Chain CLT using batch means or spectral variance methods.

Examples

```
## Bivariate Normal with mean (mu1, mu2) and covariance sigma
n <- 1e3
mu <- c(2, 50)
sigma <- matrix(c(1, 0.5, 0.5, 1), nrow = 2)
out <- BVN_Gibbs(n, mu, sigma)

out.mcse <- mcse.initseq(x = out)
out.mcse.adj <- mcse.initseq(x = out, adjust = TRUE)

# If we are only estimating the mean of the first component,
# and the second moment of the second component
g <- function(x) return(c(x[1], x[2]^2))
out.g.mcse <- mcse.initseq(x = out, g = g)
```

mcse.mat

Apply mcse to each column of the MCMC samples.

Description

Apply `mcse` to each column of the MCMC samples.

Usage

```
mcse.mat(x, size = NULL, g = NULL, method = "bm", r = 3)
```

Arguments

<code>x</code>	a matrix of values from a Markov chain of size $n \times p$.
<code>size</code>	represents the batch size in “bm” and the truncation point in “bartlett” and “tukey”. Default is NULL which implies that an optimal batch size is calculated using the <code>batchSize</code> function. Can take character values of “sqroot” and “cuberoot” or any numeric value between 1 and $n/2$. “sqroot” means size is $\lfloor n^{1/2} \rfloor$ and “cuberoot” means size is $\lfloor n^{1/3} \rfloor$.

<code>g</code>	a function such that $E(g(x))$ is the quantity of interest. The default is <code>NULL</code> , which causes the identity function to be used.
<code>method</code>	any of “bm”, “obm”, “bartlett”, “tukey”. “bm” represents batch means estimator, “obm” represents overlapping batch means estimator with, “bartlett” and “tukey” represents the modified-Bartlett window and the Tukey-Hanning windows for spectral variance estimators.
<code>r</code>	The lugsail parameters (r) that converts a lag window into its lugsail equivalent. Larger values of r will typically imply less underestimation of “cov”, but higher variability of the estimator. Default is $r = 3$ and $r = 1, 2$ are also good choices although may lead to underestimates of the variance. $r > 5$ is not recommended.

Value

`mcse.mat` returns a matrix with `ncol(x)` rows and two columns. The row names of the matrix are the same as the column names of `x`. The column names of the matrix are “est” and “se”. The j th row of the matrix contains the result of applying `mcse` to the j th column of `x`.

See Also

`mcse`, which acts on a vector.

`mcse.multi`, for a multivariate estimate of the Monte Carlo standard error.

`mcse.q` and `mcse.q.mat`, which compute standard errors for quantiles.

`mcse.multi`

Multivariate Monte Carlo standard errors for expectations.

Description

Function returns the estimate of the covariance matrix in the Markov Chain CLT using batch means or spectral variance methods (with different lag windows). The function also returns the Monte Carlo estimate.

Usage

```
mcse.multi(x, method = "bm", r = 3, size = NULL, g = NULL,
           adjust = TRUE, blather = FALSE)
```

Arguments

<code>x</code>	A matrix or data frame of Markov chain output. Number of rows is the Monte Carlo sample size.
<code>method</code>	Any of “bm”, “obm”, “bartlett”, “tukey”. “bm” represents batch means estimator, “obm” represents the overlapping batch means estimator, and “bartlett” and “tukey” represent the modified-Bartlett window and the Tukey-Hanning windows for the spectral variance estimators.

<code>r</code>	The lugsail parameters (r) that converts a lag window into its lugsail equivalent. Larger values of r will typically imply less underestimation of “cov”, but higher variability of the estimator. Default is $r = 3$ and $r = 1, 2$ are good choices. $r > 5$ is not recommended.
<code>size</code>	Represents the batch size in “bm” and the truncation point in “bartlett” and “tukey”. Default is NULL which implies that an optimal batch size is calculated using the <code>batchSize</code> function. Can take character values of “sqroot” and “cuberoot” or any numeric value between 1 and $n/2$. “sqroot” means size is $\text{floor}(n^{1/2})$ and “cuberoot” means size is $\text{floor}(n^{1/3})$.
<code>g</code>	A function that represents features of interest. g is applied to each row of x and thus g should take a vector input only. If g is NULL, g is set to be identity, which is estimation of the mean of the target density.
<code>adjust</code>	Defaults to TRUE. logical for whether the matrix should automatically be adjusted if unstable.
<code>blather</code>	If TRUE, returns under-the-hood workings of the package.

Value

A list is returned with the following components,

<code>cov</code>	a covariance matrix estimate.
<code>est</code>	estimate of $g(x)$.
<code>nsim</code>	number of rows of the input x .
<code>eigen_values</code>	eigen values of the estimate cov.
<code>method</code>	method used to calculate matrix cov.
<code>size</code>	value of size used to calculate cov.
<code>Adjustment_Used</code>	whether an adjustment was used to calculate cov.

References

Vats, D., Flegal, J. M., and, Jones, G. L Multivariate output analysis for Markov chain Monte Carlo, *Biometrika*, **106**, 321—337.

Vats, D., Flegal, J. M., and, Jones, G. L. (2018) Strong Consistency of multivariate spectral variance estimators for Markov chain Monte Carlo, *Bernoulli*, **24**, 1860—1909.

See Also

`batchSize`, which computes an optimal batch size. `mcse.initseq`, which computes an initial sequence estimator. `mcse`, which acts on a vector. `mcse.mat`, which applies mcse to each column of a matrix or data frame. `mcse.q` and `mcse.q.mat`, which compute standard errors for quantiles.

Examples

```
## Bivariate Normal with mean (mu1, mu2) and covariance sigma
n <- 1e3
mu <- c(2, 50)
sigma <- matrix(c(1, 0.5, 0.5, 1), nrow = 2)
out <- BVN_Gibbs(n, mu, sigma)

mcse.bm <- mcse.multi(x = out)
mcse.tuk <- mcse.multi(x = out, method = "tukey")

# If we are only estimating the mean of the first component,
# and the second moment of the second component

g <- function(x) return(c(x[1], x[2]^2))
mcse <- mcse.multi(x = out, g = g)
```

mcse.q

Compute Monte Carlo standard errors for quantiles.

Description

Compute Monte Carlo standard errors for quantiles.

Usage

```
mcse.q(x, q, size = NULL, g = NULL, method = c("bm", "obm", "sub"), warn = FALSE)
```

Arguments

x	a vector of values from a Markov chain.
q	the quantile of interest.
size	the batch size. The default value is “sqroot”, which uses the square root of the sample size. A numeric value may be provided if “sqroot” is not satisfactory.
g	a function such that the q th quantile of the univariate distribution function of $g(x)$ is the quantity of interest. The default is NULL, which causes the identity function to be used.
method	the method used to compute the standard error. This is one of “bm” (batch means, the default), “obm” (overlapping batch means), or “sub” (subsampling bootstrap).
warn	a logical value indicating whether the function should issue a warning if the sample size is too small (less than 1,000).

Value

mcse.q returns a list with three elements:

est	an estimate of the q th quantile of the univariate distribution function of $g(x)$.
se	the Monte Carlo standard error.
nsim	The number of samples in the input Markov chain.

References

- Flegal, J. M. (2012) Applicability of subsampling bootstrap methods in Markov chain Monte Carlo. In Wozniakowski, H. and Plaskota, L., editors, *Monte Carlo and Quasi-Monte Carlo Methods 2010* (to appear). Springer-Verlag.
- Flegal, J. M. and Jones, G. L. (2010) Batch means and spectral variance estimators in Markov chain Monte Carlo. *The Annals of Statistics*, **38**, 1034–1070.
- Flegal, J. M. and Jones, G. L. (2011) Implementing Markov chain Monte Carlo: Estimating with confidence. In Brooks, S., Gelman, A., Jones, G. L., and Meng, X., editors, *Handbook of Markov Chain Monte Carlo*, pages 175–197. Chapman & Hall/CRC Press.
- Doss, C. R., Flegal, J. M., Jones, G. L., and Neath, R. C. (2014). Markov chain Monte Carlo estimation of quantiles. *Electronic Journal of Statistics*, **8**, 2448–2478.
- Jones, G. L., Haran, M., Caffo, B. S. and Neath, R. (2006) Fixed-width output analysis for Markov chain Monte Carlo. *Journal of the American Statistical Association*, **101**, 1537–154 .

See Also

[mcse.q.mat](#), which applies `mcse.q` to each column of a matrix or data frame.

[mcse](#) and [mcse.mat](#), which compute standard errors for expectations.

Examples

```
## Bivariate Normal with mean (mu1, mu2) and covariance sigma
n <- 1e3
mu <- c(2, 50)
sigma <- matrix(c(1, 0.5, 0.5, 1), nrow = 2)
out <- BVN_Gibbs(n, mu, sigma)
x <- out[,1]

# Estimate the mean, 0.1 quantile, and 0.9 quantile with MCSEs using batch means.

mcse(x)
mcse.q(x, 0.1)
mcse.q(x, 0.9)

# Estimate the mean, 0.1 quantile, and 0.9 quantile with MCSEs using overlapping batch means.

mcse(x, method = "obm")
mcse.q(x, 0.1, method = "obm")
mcse.q(x, 0.9, method = "obm")

# Estimate E(x^2) with MCSE using spectral methods.

g <- function(x) { x^2 }
mcse(x, g = g, method = "tukey")
```

mcse.q.mat	<i>Apply mcse.q to each column of a matrix or data frame of MCMC samples.</i>
------------	---

Description

Apply `mcse.q` to each column of a matrix or data frame of MCMC samples.

Usage

```
mcse.q.mat(x, q, size = NULL, g = NULL, method = c("bm", "obm", "sub"))
```

Arguments

<code>x</code>	a matrix or data frame with each row being a draw from the multivariate distribution of interest.
<code>q</code>	the quantile of interest.
<code>size</code>	the batch size. The default value is “sqroot”, which uses the square root of the sample size. “cuberoot” will cause the function to use the cube root of the sample size. A numeric value may be provided if “sqroot” is not satisfactory.
<code>g</code>	a function such that the q th quantile of the univariate distribution function of $g(x)$ is the quantity of interest. The default is <code>NULL</code> , which causes the identity function to be used.
<code>method</code>	the method used to compute the standard error. This is one of “bm” (batch means, the default), “obm” (overlapping batch means), or “sub” (subsampling bootstrap).

Value

`mcse.q.mat` returns a matrix with `ncol(x)` rows and two columns. The row names of the matrix are the same as the column names of `x`. The column names of the matrix are “est” and “se”. The j th row of the matrix contains the result of applying `mcse.q` to the j th column of `x`.

See Also

[mcse.q](#), which acts on a vector.

[mcse](#) and [mcse.mat](#), which compute standard errors for expectations.

minESS	<i>Minimum effective sample size required for stable estimation as described in Vats et al. (2015)</i>
--------	--

Description

The function calculates the minimum effective sample size required for a specified relative tolerance level. This function can also calculate the relative precision in estimation for a given estimated effective sample size.

Usage

```
minESS(p, alpha = .05, eps = .05, ess = NULL)
```

Arguments

p	dimension of the estimation problem.
alpha	Confidence level.
eps	Tolerance level. The eps value is ignored if ess is not NULL.
ess	Estimated effective sample size. Usually the output value from multiESS.

Details

The minimum effective samples required when estimating a vector of length p , with $100(1 - \alpha)\%$ confidence and tolerance of ϵ is

$$mESS \geq \frac{2^{2/p} \pi}{(p\Gamma(p/2))^{2/p}} \frac{\chi_{1-\alpha, p}^2}{\epsilon^2}.$$

The above equality can also be used to get ϵ from an already obtained estimate of mESS.

Value

By default function returns the minimum effective sample required for a given eps tolerance. If ess is specified, then the value returned is the eps corresponding to that ess.

References

- Gong, L., and Flegal, J. M. A practical sequential stopping rule for high-dimensional Markov chain Monte Carlo. *Journal of Computational and Graphical Statistics*, **25**, 684—700.
- Vats, D., Flegal, J. M., and, Jones, G. L Multivariate output analysis for Markov chain Monte Carlo, *Biometrika*, **106**, 321—337.

See Also

[multiESS](#), which calculates multivariate effective sample size using a Markov chain and a function `g`. [ess](#) which calculates univariate effective sample size using a Markov chain and a function `g`.

Examples

```
minESS(p = 5)
```

multiESS	<i>Effective Sample Size of a multivariate Markov chain as described in Vats et al. (2015).</i>
----------	---

Description

Calculate the effective sample size of the Markov chain, using the multivariate dependence structure of the process.

Usage

```
multiESS(x, covmat = NULL, g = NULL, ...)
```

Arguments

x	a matrix or data frame of Markov chain output. Number of rows is the Monte Carlo sample size.
covmat	optional matrix estimate obtained using <code>mcse.multi</code> or <code>mcse.initseq</code> .
g	a function that represents features of interest. <code>g</code> is applied to each row of <code>x</code> and thus <code>g</code> should take a vector input only. If <code>g</code> is <code>NULL</code> , <code>g</code> is set to be identity, which is estimation of the mean of the target density.
...	arguments for <code>mcse.multi</code> function. Don't use this if a suitable matrix estimate from <code>mcse.multi</code> or <code>mcse.initseq</code> is already obtained.

Details

Effective sample size is the size of an iid sample with the same variance as the current sample. ESS is given by

$$ESS = n \frac{|\Lambda|^{1/p}}{|\Sigma|^{1/p}},$$

where Λ is the sample covariance matrix for `g` and Σ is an estimate of the Monte Carlo standard error for `g`.

Value

The function returns the estimated effective sample size.

References

Vats, D., Flegal, J. M., and, Jones, G. L Multivariate output analysis for Markov chain Monte Carlo, *Biometrika*, **106**, 321—337.

See Also

[minESS](#), which calculates the minimum effective samples required for the problem. [ess](#) which calculates univariate effective sample size using a Markov chain and a function g.

Examples

```
## Bivariate Normal with mean (mu1, mu2) and covariance sigma
n <- 1e3
mu <- c(2, 50)
sigma <- matrix(c(1, 0.5, 0.5, 1), nrow = 2)
out <- BVN_Gibbs(n, mu, sigma)

multiESS(out)
```

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