

# Package ‘freealg’

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**Version** 1.1-8  
**Maintainer** Robin K. S. Hankin <hankin.robin@gmail.com>  
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**Description** The free algebra in R with non-commuting indeterminates.  
Uses 'disordR' discipline  
(Hankin, 2022, <[doi:10.48550/ARXIV.2210.03856](https://doi.org/10.48550/ARXIV.2210.03856)>). To cite the  
package in publications please use Hankin (2022)  
<[doi:10.48550/ARXIV.2211.04002](https://doi.org/10.48550/ARXIV.2211.04002)>.  
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freealg-package	<i>The Free Algebra</i>
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**Description**

The free algebra in R with non-commuting indeterminates. Uses 'disordR' discipline (Hankin, 2022, <doi:10.48550/ARXIV.2210.03856>). To cite the package in publications please use Hankin (2022) <doi:10.48550/ARXIV.2211.04002>.

**Details**

The DESCRIPTION file:

Package:	freealg
Type:	Package
Title:	The Free Algebra
Version:	1.1-8
Authors@R:	person(given=c("Robin", "K. S."), family="Hankin", role = c("aut","cre"), email="hankin.robin@gmail.co
Maintainer:	Robin K. S. Hankin <hankin.robin@gmail.com>
Depends:	R (>= 3.5.0), methods
Description:	The free algebra in R with non-commuting indeterminates. Uses 'disordR' discipline (Hankin, 2022, <doi:
License:	GPL (>= 2)
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 BugReports: <https://github.com/RobinHankin/freealg/issues>  
 Author: Robin K. S. Hankin [aut, cre] (<<https://orcid.org/0000-0001-5982-0415>>)

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freealg-package	The Free Algebra
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inverse	Inverses
letters	Single-letter symbols
linear	A simple free algebra object
nterms	Number of terms in a freealg object
Ops.freealg	Arithmetic Ops methods for the the free algebra
pepper	Combine variables in every possible order
print.freealg	Print freealg objects
rfalg	Random free algebra objects
subs	Substitution
zero	The zero algebraic object

#### Author(s)

Robin K. S. Hankin [aut, cre] (<<https://orcid.org/0000-0001-5982-0415>>)  
 Maintainer: Robin K. S. Hankin <[hankin.robin@gmail.com](mailto:hankin.robin@gmail.com)>

#### Examples

```

a <- as.freealg("x+xyx")
b <- as.freealg("4x +XyX") # upper-case interpreted as inverse

a+b
stopifnot(a+b==b+a) # should be TRUE

a*b ==b*a # FALSE; noncommutative algebra

as.freealg("1+X+xy")^3

```

```
rfa1g()
rfa1g()^2
```

---

abelianize	<i>Abelianize a freealg object</i>
------------	------------------------------------

---

### Description

Function `abelianize()` returns a `freealg` object that is equivalent to its argument under assumption of Abelianness. The symbols are placed in alphabetical order.

### Usage

```
abelianize(x)
```

### Arguments

<code>x</code>	A <code>freealg</code> object
----------------	-------------------------------

### Details

Abelianizing a free group element means that the symbols can commute past one another. Abelianization is vectorized.

### Value

Returns an object of class `freealg`.

### Note

There is a very similar function in the **freegroup** package. However, the **frab** package is the best way to work with the free Abelian group.

### Author(s)

Robin K. S. Hankin

### Examples

```
abelianize(as.freealg("ba + 2abbba + 3abAB"))
abelianize(.[rfa1g(),rfa1g()])
```

---

accessor                      *Accessor methods for freealg objects*

---

## Description

Accessor methods for free algebra objects

## Usage

```
words(x)
coeffs(x, drop=TRUE)
coeffs(x) <- value
```

## Arguments

x	Object of class freealg
value	Numeric vector of length 1
drop	Boolean, with default TRUE meaning to drop the disord attribute, as per <code>disordR::drop()</code> and FALSE meaning to consistently return a disord object irregardless

## Details

Access or set the different parts of a freealg object. The constant term is technically a coefficient but is documented under `constant.Rd`.

“Pure” extraction and replacement (as in `a[i]` and `a[i] <- value`) is implemented experimentally. The code for extraction is cute but not particularly efficient.

## Note

There is an extended discussion of **disordR** discipline in the context of algebras in the **mvp** package at `accessor.Rd`.

## Author(s)

Robin K. S. Hankin

## See Also

[constant](#)

## Examples

```
a <- rfalg()
a
coeffs(a)
words(a) # NB: hash is identical to that of coeffs(a)

coeffs(a) <- 7 # replacement methods work
```

```
a
coeffs(a) #
```

---

adjoint

*The adjoint map*


---

### Description

The adjoint  $\text{ad}_X$  of  $X$  is a map from a Lie group  $G$  to the endomorphism group of  $G$  defined by

$$\text{ad}_X(Y) = [X, Y]$$

### Usage

```
ad(x)
```

### Arguments

x	Object nominally of class <code>freealg</code> but other classes accepted where they make sense
---	---

### Details

details here

### Note

Vignette `adjoint` gives more description

### Author(s)

Robin K. S. Hankin

### Examples

```
x <- rfalg()
y <- rfalg()

f <- ad(x)
f(y)

f(f(y)) # [x,[x,y]]
```

constant

*The constant term***Description**

Get and set the constant term of a freealg object

**Usage**

```
## S3 method for class 'freealg'
constant(x)
## S3 method for class 'numeric'
constant(x)
## S3 replacement method for class 'freealg'
constant(x) <- value
is.constant(x)
```

**Arguments**

x	Object of class freealg
value	Scalar value for the constant

**Details**

The constant term in a free algebra object is the coefficient of the empty term. In a freealg object, the map including  $\emptyset \longrightarrow v$  implies that  $v$  is the constant.

If  $x$  is a freealg object, `constant(x)` returns the value of the constant in the multivariate polynomial; if  $x$  is numeric, it returns a constant freealg object with value  $x$ .

Function `is.constant()` returns TRUE if its argument has no variables and FALSE otherwise.

Setting the coefficients of the empty freealg returns the zero (empty) object.

**Author(s)**

Robin K. S. Hankin

**Examples**

```
p <- as.freealg("1+X+Y+xy")

constant(p)
constant(p^5)

constant(p) <- 1000
p
```

---

deriv

*Differentiation of freealg objects*


---

### Description

Differentiation of freealg objects

### Usage

```
## S3 method for class 'freealg'
deriv(expr, r, ...)
```

### Arguments

expr	Object of class freealg
r	Integer vector. Elements denote variables to differentiate with respect to. If r is a character vector, it is interpreted as a=1, b=2, ..., z=26; if of length 1, "aab" is interpreted as c("a", "a", "b")
...	Further arguments, currently ignored

### Details

Experimental function `deriv(S, v)` returns  $\frac{\partial^r S}{\partial v_1 \partial v_2 \dots \partial v_r}$ . The Leibniz product rule

$$(u \cdot v)' = uv' + u'v$$

operates even if (as here)  $u, v$  do not commute. For example, if we wish to differentiate  $aaba$  with respect to  $a$ , we would write  $f(a) = aaba$  and then

$$f(a + \delta a) = (a + \delta a)(a + \delta a)b(a + \delta a)$$

and working to first order we have

$$f(a + \delta a) - f(a) = (\delta a)aba + a(\delta a)ba + aab(\delta a).$$

In the package:

```
> deriv(as.freealg("aaba"), "a")
free algebra element algebraically equal to
+ 1*aab(da) + 1*a(da)ba + 1*(da)aba
```

A term of a freealg object can include negative values which correspond to negative powers of variables. Thus:



```
> deriv(as.freealg("AAAA"), "a")
free algebra element algebraically equal to
- 1*AAAA(da)A - 1*AAA(da)AA - 1*AA(da)AAA - 1*A(da)AAAA
```

(see also the examples). Vector  $r$  may include negative integers which mean to differentiate with respect to the inverse of the variable:

```
> deriv(as.freealg("3abcbCC"), "C")
free algebra element algebraically equal to
+ 3*abcbC(dC) + 3*abcb(dC)C - 3*abc(dC)cbCC
```

It is possible to perform repeated differentiation by passing a suitable value of  $r$ . For  $\frac{\partial^2}{\partial a \partial c}$ :

```
> deriv(as.freealg("aaabAcx"), "ac")
free algebra element algebraically equal to
- 1*aaabA(da)A(dc)x + 1*aa(da)bA(dc)x + 1*a(da)abA(dc)x + 1*(da)aabA(dc)x
```

The infinitesimal indeterminates (“da” etc) are represented by  $\text{SHRT\_MAX}+r$ , where  $r$  is the integer for the symbol, and  $\text{SHRT\_MAX}$  is the maximum short integer. This includes negative  $r$ . So the maximum number for any symbol is  $\text{SHRT\_MAX}$ . Inverse elements such as  $A$ , being represented by negative integers, have differentials that are  $\text{SHRT\_MAX}-r$ .

Function `deriv()` calls helper function `lowlevel_diffn()` which is documented at `Ops.freealg.Rd`.

A vignette illustrating this concept and furnishing numerical verification of the code in the context of matrix algebra is given at `inst/freealg_matrix.Rmd`.

## Author(s)

Robin K. S. Hankin

## Examples

```
deriv(as.freealg("4*aaaabaacAc"), 1)

x <- rfa1g()
deriv(x, 1:3)

y <- rfa1g(7, 7, 17, TRUE)

deriv(y, 1:5) - deriv(y, sample(1:5)) # should be zero
```

dot-class

Class “dot”

**Description**

The dot object is defined so that `.[x,y]` returns the commutator of `x` and `y`, that is,  $xy-yx$  or the Lie bracket  $[x,y]$ . It would have been nice to use `[x,y]` (that is, without the dot) but although this is syntactically consistent, it cannot be done in R.

The “meat” of the dot functionality is:

```
setClass("dot", slots = c(ignore='numeric'))
`.` <- new("dot")
setMethod("[", signature(x="dot", i="ANY", j="ANY"), function(x,i,j,drop){i*j-j*i})
```

The package code includes other bits and pieces such as informative error messages for idiom such as `.[]`. The package defines a matrix method for the dot object. This is because “`*`” returns (incorrectly, in my view) the elementwise product, not the matrix product.

The Jacobi identity, satisfied by any associative algebra, is

$$[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$$

and the left hand side is returned by `jacobi()`, which should be zero (for some definition of “zero”).

Function `ad()` returns the adjoint operator. The adjoint vignette provides details and examples of the adjoint operator.

The dot object is generated by running script `inst/dot.Rmd`, which includes some further discussion and technical documentation, and creates file `dot.rda` which resides in the `data/` directory.

**Value**

Always returns an object of the same class as `xy`.

**Slots**

`ignore`: Object of class “numeric”, just a formal placeholder

**Methods**

```
[ signature(x = "dot", i = "ANY", j = "ANY"): ...
[ signature(x = "dot", i = "ANY", j = "missing"): ...
[ signature(x = "dot", i = "function", j = "function"): ...
[ signature(x = "dot", i = "matrix", j = "matrix"): ...
[ signature(x = "dot", i = "missing", j = "ANY"): ...
[ signature(x = "dot", i = "missing", j = "missing"): ...
```

**Author(s)**

Robin K. S. Hankin

**Examples**

```

.[as.freealg("x"),as.freealg("y")]
.[as.freealg("x"),as.freealg("y+2z")]
.[as.freealg("x+y+2xYx"),as.freealg("x+y+2xYx")]

x <- rfalg()
y <- rfalg()
z <- rfalg()

jacobi(x,y,z) # Jacobi identity
.[x,.[y,z]] + .[y,.[z,x]] + .[z,.[x,y]] # Jacobi, expanded

f <- ad(x)
f(y)

rM <- function(...){matrix(sample(1:9,9),3,3)} # a random matrix

M <- rM()
N <- rM()
O <- rM()

.[M,N]
jacobi(M,N,O)

plot(.[sin,tan](seq(from=0,to=1,len=100)))

```

---

drop

---

*Drop redundant information*


---

**Description**

Coerce constant free algebra objects to numeric

**Usage**

drop(x)

**Arguments**

x                      Free algebra object

**Details**

If its argument is a constant freealg object, coerce to numeric. Modelled on `base::drop()`.

**Note**

A few functions in the package take `drop` as an argument which, if `TRUE`, means that the function returns a dropped value.

**Author(s)**

Robin K. S. Hankin

**See Also**

[constant](#), [coeffs](#)

**Examples**

```
drop(linear(1:5))
drop(4+linear(1:5)*0)
```

---

freealg

*The free algebra*

---

**Description**

Create, test for, and coerce to, freealg objects

**Usage**

```
freealg(words, coeffs)
is_ok_free(words, coeffs)
is.freealg(x)
as.freealg(x,...)
char_to_freealg(ch)
natural_char_to_freealg(string)
string_to_freealg(string)
vector_to_free(v, coeffs)
```

**Arguments**

<code>words</code>	Terms of the algebra object, eg <code>c(1, 2, -1, -3, -2)</code> corresponds to <code>abACB</code> because $a = 1$ , $b = 2$ etc; uppercase, or negative number, means inverse
<code>coeffs</code>	Numeric vector corresponding to the coefficients of each element of the word list
<code>string</code>	Character string
<code>ch</code>	Character vector

<code>v</code>	Vector of integers
<code>x</code>	Object possibly of class <code>freealg</code>
<code>...</code>	Further arguments, passed to the methods

## Details

Function `freealg()` is the formal creation mechanism for `freealg` objects. However, it is not very user-friendly; it is better to use `as.freealg()` in day-to-day use (although it does use heuristics for the coefficients if not supplied).

Low-level helper function `is_ok_freealg()` checks for consistency of its arguments.

A `freealg` object is a two-element list. The first element is a list of integer vectors representing the indices and the second is a numeric vector of coefficients. Thus, for example:

```
> as.freealg("a+4bd+3abbbbc")
free algebra element algebraically equal to
+ 1*a + 3*abbbbc + 4*bd
> dput(as.freealg("a+4bd+3abbbbc"))
structure(list(indices = list(1L, c(1L, 2L, 2L, 2L, 3L),
                             c(2L, 4L)), coeffs = c(1, 3, 4)), class = "freealg")
```

Observe that the order of the terms is not preserved and indeed is undefined (implementation-specific). Zero entries are stripped out.

Character strings may be coerced to `freealg` objects; `as.freealg()` calls `natural_char_to_freealg()`, which is user-friendly. Functions `char_to_freealg()` and `string_to_freealg()` are low-level helper functions. These functions assume that upper-case letters are the multiplicative inverses of the lower-case equivalents; so for example `as.freealg("aA")` and `as.freealg(aBcCbA)` evaluate to one. This can be confusing with the default print method.

## Note

Internally, the package uses signed integers and as such can have `.Machine$integer.max` different symbols; on my machine this is 2147483647. Of course the print method cannot deal with this as it only has 26 symbols for letters a-z (and A-Z for the inverses), but the objects themselves do not care about the print method. Note also that the experimental calculus facility (as per `deriv()`) reserves numbers in the range  $\text{SHRT\_MAX} \pm r$  for infinitesimals, where  $r$  is the integer for a symbol. This system might change in the future.

## Author(s)

Robin K. S. Hankin

## Examples

```
freealg(list(1:2, 2:1,numeric(0),1:6),1:4)
freealg(list(1:2, 2:1,numeric(0),1:6)) # heuristics for coeffs: assume 1

freealg(sapply(1:5,seq_len),1:5)
```

```
freealg(replicate(5,sample(-5:5,rgeom(1,1/5),replace=TRUE)),1:5)

as.freealg("1+xaX")^5
```

---

freealg-class

Class “freealg”

---

### Description

The formal S4 class for freealg objects

### Objects from the Class

Formal class *freealg* is used for functions such as drop() which need a S4 object.

### Author(s)

Robin K. S. Hankin

---

grade

The grade (or degree) of terms in a freealg object

---

### Description

The free algebra  $\mathcal{B}$  is a *graded* algebra: that is, for each integer  $n \geq 0$  there is a homogeneous subspace  $\mathcal{B}_n$  with  $\mathcal{B}_0 = \mathcal{R}$  and

$$\mathcal{B} = \bigoplus_{n=0}^{\infty} \mathcal{B}_n, \quad \text{and} \quad \mathcal{B}_n \mathcal{B}_m \subseteq \mathcal{B}_{n+m} \quad \text{for all } m, n \geq 0.$$

The elements of  $\cup_{n \geq 0} \mathcal{B}_n$  are called *homogeneous* and those of  $\mathcal{B}_n$  are called homogenous of degree (or grade)  $n$ .

The *grade* of a term is the number of symbols in it. Thus the grade of xxx and 4xxy is 3; the grade of a constant is zero. Because the terms are stored in an implementation-specific way, the grade of a multi-term object is a disord object.

The grade of the zero freealg object, grade(as.freealg(0)), is defined to be  $-\infty$ , as per Knuth [TAOCP, volume 2, p436]. This ensures that  $\max(\text{grades}(\text{abelianize}(x))) \leq \max(\text{grades}(x))$  is always satisfied. However, a case for NULL could be made.

**Usage**

```

grades(x)
grade(x,n)
grade(x,n) <- value
deg(x)

```

**Arguments**

x	Freealg object
n	Integer vector
value	Replacement value, a numeric vector

**Details**

grades(x) returns the grade (number of symbols) in each term of a freealg object x.

deg(x) returns the maximum of the grades of each symbol of x; max(grades(x)).

grade(x,n) returns the freealg object comprising terms with grade n (which may be a vector). Note that this function is considerably less efficient than `clifford::grade()`.

grade(x,n) <- value sets the coefficients of terms with grade n. For value, a length-one numeric vector is accepted (notably zero, which kills terms of grade n) and also a freealg object comprising terms of grade n.

**Value**

Returns a `disord` object

**Note**

A similar concept *grade* is discussed in the **clifford** package

**Author(s)**

Robin K. S. Hankin

**References**

H. Munthe-Kaas and B. Owren 1999. "Computations in a free Lie algebra", *Phil. Trans. R. Soc. Lond. A*, 357:957–981 (theorem 3.8)

**Examples**

```

X <- as.freealg("1 -x + 5*y + 6*x*y -8*x*x*x*x*y*x")
X
grades(X)

a <- rfalg(30)
a
grades(a)

```

```

grade(a,2)
grade(a,2) <- 0 # kill all grade-2 terms
a

grade(a,1) <- grade(a,1) * 888
a

```

---

horner

*Horner's method*


---

## Description

Horner's method for multivariate polynomials

## Usage

```
horner(P,v)
```

## Arguments

P	Free algebra polynomial
v	Numeric vector of coefficients

## Details

This function is (almost) the same as `mvp:horner()`.

Given a polynomial

$$p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$$

it is possible to express  $p(x)$  in the algebraically equivalent form

$$p(x) = a_0 + x(a_1 + x(a_2 + \cdots + x(a_{n-1} + xa_n)\cdots))$$

which is much more efficient for evaluation, as it requires only  $n$  multiplications and  $n$  additions, and this is optimal. Function `horner()` will take a `freealg` object for its first argument.

## Author(s)

Robin K. S. Hankin

## Examples

```

horner("x", 1:4) # note constant term is 1.

horner("x+y",1:3) # note presence of xy and yx terms

horner("1+x+xyX",1:3)

```



inverse

*Inverses***Description**

Multiplicative inverses of symbols in the free algebra

**Usage**

```
all_pos(x)
keep_pos(x)
```

**Arguments**

x                      Freealg object

**Details**

Function `all_pos()` tests for its argument having only positive powers (that is, no inverse symbols present); function `keep_pos()` discards any term with a negative power.

At various points in the package, it is assumed that upper-case letters are the multiplicative inverses of the lower-case equivalents; so for example `as.freealg("aA")` and `as.freealg("aBcCbA")` evaluate to one. This can be confusing with the default print method.

Even though individual symbols have multiplicative inverses, a general element of the free algebra will not have a multiplicative inverse. For example,  $1+x$  does not have an inverse. The free algebra is not a division algebra, in general.

**Author(s)**

Robin K. S. Hankin

**Examples**

```
all_pos(rfalg(include.negative = TRUE))
all_pos(rfalg(include.negative = FALSE))
```

```
as.freealg("1+xaX")^5
```

---

letters	<i>Single-letter symbols</i>
---------	------------------------------

---

### Description

Variables  $a, b, \dots, z$  and their inverses A-Z are given their freealg semantic meaning.

### Details

Sometimes it is convenient in an R session to have all 26 letters a-z and all 26 uppercase letters A-Z adopt their free algebra interpretations. To access this, load the lettersymbols dataset, which is provided with the package in the inst directory:

```
load(system.file("lettersymbols.rda", package="freealg"))
```

Executing this allows you to do cool things such as the following:

```
> (1+a-b^2)^4
free algebra element algebraically equal to
+ 1 + 4a + 6aa + 4aaa + aaaa - aaabb - 4aabb - aabba + aabbbb - 6abb - 4abba -
abbaa + abbabb + 4abbbb + abbbba - abbbbbb - 4bb - 6bba - 4bbaa - bbaaa +
bbaabb + 4bbabb + bbabba - bbabbbb + 6bbbb + 4bbbba + bbbbaa - bbbbab -
4bbbbb - bbbbbb + bbbbbb
```

Lowercase letters c, q, t, and uppercase letters C, D, F, I, T might pose difficulties.

These objects can also be generated by running script inst/symb.Rmd, which includes some further discussion and technical documentation and creates file lettersymbols.rda which formerly resided in the data/ directory.

### Author(s)

Robin K. S. Hankin

---

linear	<i>A simple free algebra object</i>
--------	-------------------------------------

---

### Description

Create simple free algebra objects including linear expressions. For example:

```
> linear(1:3)
free algebra element algebraically equal to
+ 1*a + 2*b + 3*c
> linear(1:3,power=5)
free algebra element algebraically equal to
+ 1*aaaaa + 2*bbbbb + 3*ccccc
```

### Usage

```
linear(x,power=1)
```

### Arguments

x	Numeric vector of terms
power	Integer vector of powers

### Note

It is instructive to compare the functionality documented here with their **mvp** equivalents. Many of the functions documented at `mvp::special.Rd` do not make sense in the context of the free algebra. Function `mvp::product()`, for example, imposes an order on the expansion.

Function `constant()` is documented at `constant.Rd`, but is listed below for convenience.

### Author(s)

Robin K. S. Hankin

### See Also

[constant](#), [zero](#)

### Examples

```
linear(1:3)
linear(1:3,power=5)
linear(1:3,power=3:1)
```

---

nterms

*Number of terms in a freealg object*


---

### Description

Number of terms in a freealg object; number of coefficients

### Usage

```
nterms(x)
```

### Arguments

x	Freealg object
---	----------------

### Value

Returns a non-negative integer

**Author(s)**

Robin K. S. Hankin

**Examples**

```
(a <- freealg(list(1:3,4:7,1:10),1:3))
nterms(a)
nterms(a+1)
nterms(a*0)
```

Ops.freealg

*Arithmetic Ops methods for the the free algebra***Description**

Arithmetic operators for manipulation of freealg objects such as addition, multiplication, powers, etc

**Usage**

```
## S3 method for class 'freealg'
Ops(e1, e2)
free_negative(S)
free_power_scalar(S,n)
free_eq_free(e1,e2)
free_plus_numeric(S,x)
free_plus_free(e1,e2)
lowlevel_simplify(words,coeffs)
lowlevel_free_prod(words1,coeffs1,words2,coeffs2)
lowlevel_free_sum(words1,coeffs1,words2,coeffs2)
lowlevel_free_power(words,coeffs,n)
lowlevel_diffn(words,coeffs,r)
lowlevel_subs(words1, coeffs1, words2, coeffs2, r)
inv(S)
```

**Arguments**

S, e1, e2	Objects of class freealg
n	Integer, possibly non-positive
r	Integer vector indicating variables to differentiate with respect to
x	Scalar value
words, words1, words2	A list of words, that is, a list of integer vectors representing the variables in each term
coeffs, coeffs1, coeffs2	Numeric vector representing the coefficients of each word

## Details

The function `Ops.freealg()` passes binary arithmetic operators (“+”, “-”, “\*”, “^”, and “==”) to the appropriate specialist function.

The caret, as in  $a^n$ , denotes arithmetic exponentiation, as in  $x^3 == x*x*x$ . As an experimental feature, this is (sort of) vectorised: if  $n$  is a vector, then  $a^n$  returns the sum of  $a$  raised to the power of each element of  $n$ . For example,  $a^c(n1, n2, n3)$  is  $a^{n1} + a^{n2} + a^{n3}$ . Internally,  $n$  is tabulated in the interests of efficiency, so  $a^c(0, 2, 5, 5, 5, ) = 1 + a^2 + 3a^5$  is evaluated with only a single fifth power. Similar functionality is implemented in the **mvp** package.

The only comparison operators are equality and inequality;  $x == y$  is defined as `is.zero(x-y)`.

Functions `lowlevel_foo()` are low-level functions that interface directly with the C routines in the `src/` directory and are not intended for the end-user.

Function `inv()` is defined only for `freealg` objects with a single term. If  $x$  has a single term we have  $\text{inv}(x)*x = x*\text{inv}(x) = 1$ . There is no corresponding division in the package because  $a/b$  may be either  $a*\text{inv}(b)$  or  $\text{inv}(b)*a$ .

## Author(s)

Robin K. S. Hankin

## Examples

```
rfalg()
as.freealg("1+x+xy+yx") # variables are non-commutative
as.freealg("x") * as.freealg("X") # upper-case letters are lower-case inverses

constant(as.freealg("x+y+X+Y")^6) # OEIS sequence A035610

inv(as.freealg("2aaabAAAx"))

as.freealg("a")^(1:7)
```

---

pepper

*Combine variables in every possible order*

---

## Description

Given a list of variables, construct every term comprising only those variables; function `pepper()` returns a free algebra object equal to the sum of these terms.

The function is named for a query from an exam question set by Sarah Marshall in which she asked how many ways there are to arrange the letters of word “pepper”, the answer being  $\binom{6}{1\,2\,3} = \frac{6!}{1!2!3!} = 60$ .

Function `multiset()` in the **partitions** package gives related functionality; for the record, one way to reproduce `pepper("pepper")` would be

```
apply(matrix(c("p", "e", "r")[multiset(c(1, 1, 1, 2, 2, 3))], nrow=6), 2, paste, collapse="")
```

**Usage**

pepper(v)

**Arguments**

v                      Variables to combine. If a character string, coerce to variable numbers

**Author(s)**

Robin K. S. Hankin

**See Also**

[linear](#)

**Examples**

```
pepper(c(1,1,1,1,1,1,2)) # 6 a's and 1 b
pepper(c(1,2,2,2,3))      # 1 a, 3 b's and 1 c
pepper("pepper")
```

---

print	<i>Print freealg objects</i>
-------	------------------------------

---

**Description**

Print methods for free algebra objects. The indeterminates are represented using lowercase letters a-z (currently hard coded).

**Usage**

```
## S3 method for class 'freealg'
print(x,...)
```

**Arguments**

x                      Object of class freealg in the print method  
...                    Further arguments, currently ignored

**Note**

The print method does not change the internal representation of a freealg object, which is a two-element list, the first of which is a list of integer vectors representing words, and the second is a numeric vector of coefficients.

The print method uses lowercase letters a-z to represent the indeterminates; this is currently hard coded:

```

> (x <- as.freealg("6abbbc + 7cax"))
free algebra element algebraically equal to
+ 6*abbbc + 7*cax
> unclass(x)
$indices
$indices[[1]]
[1] 1 2 2 2 3

$indices[[2]]
[1] 3 1 24

$coeffs
[1] 6 7

```

The print method has special dispensation for length-zero freealg objects but these are not handled entirely consistently.

The print method is sensitive to the value of `getOption("usecaret")`, defaulting to "FALSE". The default is to use uppercase letters to represent multiplicative inverses. Thus, the inverse of `a` appears as either "`a^-1`" if `usecaret` is TRUE, and "`A`" if FALSE. Carets become cumbersome for powers above the first. For example, the default notation for  $aba^{-2}$  is `abAA` but becomes `aba^-1a^-1` if `usecaret` is TRUE.

The symbols for the indeterminates are currently hardcoded as `c(letters, LETTERS)`. The intent is to be able to signify 52 distinct indeterminates, `a-z, A-Z`. This works fine if option `usecaret` is TRUE. But if option `usecaret` is FALSE, this can be confusing: for example, indeterminate number 1 appears as `a`, and its inverse would appear as "`A`". But indeterminate number 27 also appears as "`A`". They look the same, but no warning is given: caveat emptor!

The method is also sensitive to `getOption("mulsym")`, defaulting to NULL. This is the multiplication symbol used between the coefficient and the indeterminate string. Sometimes an asterisk, `*` or a space, might be useful. If `mulsym` takes its default of NULL [or a length zero string], the print method suppresses coefficients of  $\pm 1$ .

Integers exceeding `SHRT_MAX` are reserved for infinitesimals, which are printed as "`da`"; see the note at `deriv.Rd` for details.

### Author(s)

Robin K. S. Hankin

### See Also

[freealg](#), [deriv](#)

### Examples

```

rfalg()

x <- rfalg(inc=TRUE)
x                                     # default

```

```

options("usecaret" = TRUE) # use caret
x
options("usecaret" = FALSE) # back to the default
x

x <- freealg(list(5,1:4,3,8,7),c(1,1,1,3,22))
x

options(mulsym = "*")
x
options(mulsym = NULL) # restore default

```

---

rfalg

*Random free algebra objects*


---

## Description

Random elements of the free algebra, intended as quick “get you going” examples of freealg objects

## Usage

```

rfalg(n=7, distinct=3, maxsize=4, include.negative=FALSE)
rfalgg(n=30, distinct=8, maxsize=7, include.negative=FALSE)
rfalggg(n=100, distinct=26, maxsize=30, include.negative=FALSE)

```

## Arguments

n	Number of terms to generate
distinct	Number of distinct symbols to use
maxsize	Maximum number of symbols in any word
include.negative	Boolean, with default FALSE meaning to use only positive symbols (lower-case letters) and TRUE meaning to use upper-case letters as well, corresponding to the inverse of the lower-case symbols

## Details

What you see is what you get, basically. A term such as aaBaAbaC will be simplified to aaaC.

Functions rfalgg() and rfalggg() return successively more complicated freealg objects.

## Author(s)

Robin K. S. Hankin



**Examples**

```
rfalg()
rfalg(include.negative=TRUE)^2

constant(rfalg())
```

---

subs	<i>Substitution</i>
------	---------------------

---

**Description**

Substitute symbols in a freealg object for numbers or other freealg objects

**Usage**

```
subs(S, ...)
subsu(S1, S2, r)
```

**Arguments**

S, S1, S2	Objects of class freealg
r	Integer specifying symbol to substitute ( $a = 1, b = 2$ etc)
...	named arguments corresponding to variables to substitute

**Details**

Function subs() substitutes variables for freealg objects (coerced if necessary) using natural R idiom. Observe that this type of substitution is sensitive to order:

```
> subs("ax", a="1+x", x="1+a")
free algebra element algebraically equal to
+ 2 + 3*a + 1*aa
```

```
> subs("ax", x="1+a", a="1+x")
free algebra element algebraically equal to
+ 2 + 3*x + 1*xx
```

Functions subsu() is a lower-level formal function, not really intended for the end-user. Function subsu() takes S1 and substitutes occurrences of symbol r with S2.

No equivalent to `mvp : subvec()` is currently implemented.

**Value**

Returns a freealg object.

**Note**

Function `subs()` is one place in the package where the use of letters is effectively hard-wired in. Idiom such as

```
subs("abccc",b="1+3x")
```

is very nice, but identifies “b” with 2. Note that argument `r` of `subs()` is canonically an integer but a single character is interpreted as a letter. See also the note at `freealg.Rd`.

**Author(s)**

Robin K. S. Hankin

**Examples**

```
subs("abccc",b="1+3x")
subs("aaaa",a="1+x") # binomial
```

```
subs("abA",b=31)
```

```
subs("1+a",a="A") # can substitute for an inverse
subs("A",a="1+x") # inverses are not substituted for
```

```
## Sequential substitution works:
```

```
subs("abccc",b="1+3x",x="1+d+2e")
subs(rfalg(),a=rfalg())
```

---

zero

*The zero algebraic object*

---

**Description**

Test for a `freealg` object's being zero

**Usage**

```
is.zero(x)
```

**Arguments**

`x`                      Object of class `freealg`

**Details**

Function `is.zero()` returns TRUE if `x` is indeed the zero free algebra object. It is defined as `length(coeffs(x))==0` for reasons of efficiency, but conceptually it returns `x==constant(0)`.

(Use `constant(0)` to create the zero object).

**Author(s)**

Robin K. S. Hankin

**See Also**

[constant](#)

**Examples**

```
stopifnot(is.zero(constant(0)))
```

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