

Package ‘crch’

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quarto

Description Different approaches to censored or truncated regression with conditional heteroscedasticity are provided. First, continuous distributions can be used for the (right and/or left censored or truncated) response with separate linear predictors for the mean and variance. Second, cumulative link models for ordinal data (obtained by interval-censoring continuous data) can be employed for heteroscedastic extended logistic regression (HXLRL). In the latter type of models, the intercepts depend on the thresholds that define the intervals. Infrastructure for working with censored or truncated normal, logistic, and Student-t distributions, i.e., d/p/q/r functions and distributions3 objects.

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CensoredLogistic	Create a Censored Logistic Distribution
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Description

Class and methods for left-, right-, and interval-censored logistic distributions using the workflow from the **distributions3** package.

Usage

CensoredLogistic(location = 0, scale = 1, left = -Inf, right = Inf)

Arguments

location	numeric. The location parameter of the underlying <i>uncensored</i> logistic distribution, typically written μ in textbooks. Can be any real number, defaults to 0.
scale	numeric. The scale parameter (standard deviation) of the underlying <i>uncensored</i> logistic distribution, typically written σ in textbooks. Can be any positive number, defaults to 1.
left	numeric. The left censoring point. Can be any real number, defaults to $-\text{Inf}$ (uncensored). If set to a finite value, the distribution has a point mass at <i>left</i> whose probability corresponds to the cumulative probability function of the <i>uncensored</i> logistic distribution at this point.
right	numeric. The right censoring point. Can be any real number, defaults to Inf (uncensored). If set to a finite value, the distribution has a point mass at <i>right</i> whose probability corresponds to 1 minus the cumulative probability function of the <i>uncensored</i> logistic distribution at this point.

Details

The constructor function `CensoredLogistic` sets up a distribution object, representing the censored logistic probability distribution by the corresponding parameters: the latent mean location $= \mu$ and latent standard deviation $\text{scale} = \sigma$ (i.e., the parameters of the underlying *uncensored* logistic variable), the left censoring point (with $-\text{Inf}$ corresponding to uncensored), and the right censoring point (with Inf corresponding to uncensored).

The censored logistic distribution has probability density function (PDF) $f(x)$:

$$\begin{array}{ll} \Lambda((\text{left} - \mu)/\sigma) & \text{if } x \leq \text{left} \\ 1 - \Lambda((\text{right} - \mu)/\sigma) & \text{if } x \geq \text{right} \\ \lambda((x - \mu)/\sigma)/\sigma & \text{otherwise} \end{array}$$

where Λ and λ are the cumulative distribution function and probability density function of the standard logistic distribution, respectively.

All parameters can also be vectors, so that it is possible to define a vector of censored logistic distributions with potentially different parameters. All parameters need to have the same length or must be scalars (i.e., of length 1) which are then recycled to the length of the other parameters.

For the `CensoredLogistic` distribution objects there is a wide range of standard methods available to the generics provided in the **distributions3** package: `pdf` and `log_pdf` for the (log-)density (PDF), `cdf` for the probability from the cumulative distribution function (CDF), `quantile` for quantiles, `random` for simulating random variables, `crps` for the continuous ranked probability score (CRPS), and `support` for the support interval (minimum and maximum). Internally, these methods rely on the usual d/p/q/r functions provided for the censored logistic distributions in the **crch** package, see `dclgis`, and the `crps_clogis` function from the **scoringRules** package. The methods `is_discrete` and `is_continuous` can be used to query whether the distributions are discrete on the entire support (always FALSE) or continuous on the entire support (only TRUE if there is no censoring, i.e., if both *left* and *right* are infinite).

See the examples below for an illustration of the workflow for the class and methods.

Value

A CensoredLogistic distribution object.

See Also

[dclogis](#), [Logistic](#), [TruncatedLogistic](#), [CensoredNormal](#), [CensoredStudentsT](#)

Examples

```
## package and random seed
library("distributions3")
set.seed(6020)

## three censored logistic distributions:
## - uncensored standard logistic
## - left-censored at zero with latent location = 1 and scale = 1
## - interval-censored in [0, 5] with latent location = 2 and scale = 1
X <- CensoredLogistic(
  location = c( 0, 1, 2),
  scale    = c( 1, 1, 1),
  left     = c(-Inf, 0, 0),
  right    = c( Inf, Inf, 5)
)

X

## compute mean of the censored distribution
mean(X)
## higher moments (variance, skewness, kurtosis) are not implemented yet

## support interval (minimum and maximum)
support(X)

## simulate random variables
random(X, 5)

## histograms of 1,000 simulated observations
x <- random(X, 1000)
hist(x[1, ], main = "uncensored")
hist(x[2, ], main = "left-censored at zero")
hist(x[3, ], main = "interval-censored in [0, 5]")

## probability density function (PDF) and log-density (or log-likelihood)
x <- c(0, 0, 1)
pdf(X, x)
pdf(X, x, log = TRUE)
log_pdf(X, x)

## cumulative distribution function (CDF)
cdf(X, x)
```

```
## quantiles
quantile(X, 0.5)

## cdf() and quantile() are inverses (except at censoring points)
cdf(X, quantile(X, 0.5))
quantile(X, cdf(X, 1))

## all methods above can either be applied elementwise or for
## all combinations of X and x, if length(X) = length(x),
## also the result can be assured to be a matrix via drop = FALSE
p <- c(0.05, 0.5, 0.95)
quantile(X, p, elementwise = FALSE)
quantile(X, p, elementwise = TRUE)
quantile(X, p, elementwise = TRUE, drop = FALSE)

## compare theoretical and empirical mean from 1,000 simulated observations
cbind(
  "theoretical" = mean(X),
  "empirical" = rowMeans(random(X, 1000))
)

## evaluate continuous ranked probability score (CRPS) using scoringRules
library("scoringRules")
crps(X, x)
```

CensoredNormal

Create a Censored Normal Distribution

Description

Class and methods for left-, right-, and interval-censored normal distributions using the workflow from the **distributions3** package.

Usage

```
CensoredNormal(mu = 0, sigma = 1, left = -Inf, right = Inf)
```

Arguments

mu	numeric. The location parameter of the underlying <i>uncensored</i> normal distribution, typically written μ in textbooks. Can be any real number, defaults to 0.
sigma	numeric. The scale parameter (standard deviation) of the underlying <i>uncensored</i> normal distribution, typically written σ in textbooks. Can be any positive number, defaults to 1.
left	numeric. The left censoring point. Can be any real number, defaults to $-\text{Inf}$ (uncensored). If set to a finite value, the distribution has a point mass at left whose probability corresponds to the cumulative probability function of the <i>uncensored</i> normal distribution at this point.

right numeric. The right censoring point. Can be any real number, defaults to Inf (uncensored). If set to a finite value, the distribution has a point mass at **right** whose probability corresponds to 1 minus the cumulative probability function of the *uncensored* normal distribution at this point.

Details

The constructor function `CensoredNormal` sets up a distribution object, representing the censored normal probability distribution by the corresponding parameters: the latent mean $\mu = \mu$ and latent standard deviation $\sigma = \sigma$ (i.e., the parameters of the underlying *uncensored* normal variable), the left censoring point (with $-\text{Inf}$ corresponding to uncensored), and the right censoring point (with Inf corresponding to uncensored).

The censored normal distribution has probability density function (PDF) $f(x)$:

$$\begin{aligned} &\Phi((\text{left} - \mu)/\sigma) && \text{if } x \leq \text{left} \\ &1 - \Phi((\text{right} - \mu)/\sigma) && \text{if } x \geq \text{right} \\ &\phi((x - \mu)/\sigma)/\sigma && \text{otherwise} \end{aligned}$$

where Φ and ϕ are the cumulative distribution function and probability density function of the standard normal distribution respectively.

All parameters can also be vectors, so that it is possible to define a vector of censored normal distributions with potentially different parameters. All parameters need to have the same length or must be scalars (i.e., of length 1) which are then recycled to the length of the other parameters.

For the `CensoredNormal` distribution objects there is a wide range of standard methods available to the generics provided in the **distributions3** package: `pdf` and `log_pdf` for the (log-)density (PDF), `cdf` for the probability from the cumulative distribution function (CDF), `quantile` for quantiles, `random` for simulating random variables, `crps` for the continuous ranked probability score (CRPS), and `support` for the support interval (minimum and maximum). Internally, these methods rely on the usual d/p/q/r functions provided for the censored normal distributions in the **crch** package, see `dcnorm`, and the `crps_cnorm` function from the **scoringRules** package. The methods `is_discrete` and `is_continuous` can be used to query whether the distributions are discrete on the entire support (always FALSE) or continuous on the entire support (only TRUE if there is no censoring, i.e., if both `left` and `right` are infinite).

See the examples below for an illustration of the workflow for the class and methods.

Value

A `CensoredNormal` distribution object.

See Also

`dcnorm`, `Normal`, `TruncatedNormal`, `CensoredLogistic`, `CensoredStudentsT`

Examples

```
## package and random seed
library("distributions3")
set.seed(6020)
```

```

## three censored normal distributions:
## - uncensored standard normal
## - left-censored at zero (Tobit) with latent mu = 1 and sigma = 1
## - interval-censored in [0, 5] with latent mu = 1 and sigma = 2
X <- CensoredNormal(
  mu    = c( 0, 1, 1),
  sigma = c( 1, 1, 2),
  left  = c(-Inf, 0, 0),
  right = c( Inf, Inf, 5)
)

X

## compute mean and variance of the censored distribution
mean(X)
variance(X)
## higher moments (skewness, kurtosis) are not implemented yet

## support interval (minimum and maximum)
support(X)

## simulate random variables
random(X, 5)

## histograms of 1,000 simulated observations
x <- random(X, 1000)
hist(x[1, ], main = "uncensored")
hist(x[2, ], main = "left-censored at zero")
hist(x[3, ], main = "interval-censored in [0, 5]")

## probability density function (PDF) and log-density (or log-likelihood)
x <- c(0, 0, 1)
pdf(X, x)
pdf(X, x, log = TRUE)
log_pdf(X, x)

## cumulative distribution function (CDF)
cdf(X, x)

## quantiles
quantile(X, 0.5)

## cdf() and quantile() are inverses (except at censoring points)
cdf(X, quantile(X, 0.5))
quantile(X, cdf(X, 1))

## all methods above can either be applied elementwise or for
## all combinations of X and x, if length(X) = length(x),
## also the result can be assured to be a matrix via drop = FALSE
p <- c(0.05, 0.5, 0.95)
quantile(X, p, elementwise = FALSE)

```

```

quantile(X, p, elementwise = TRUE)
quantile(X, p, elementwise = TRUE, drop = FALSE)

## compare theoretical and empirical mean from 1,000 simulated observations
cbind(
  "theoretical" = mean(X),
  "empirical" = rowMeans(random(X, 1000))
)

## evaluate continuous ranked probability score (CRPS) using scoringRules
library("scoringRules")
crps(X, x)

```

CensoredStudentsT	Create a Censored Student's T Distribution
-------------------	--

Description

Class and methods for left-, right-, and interval-censored t distributions using the workflow from the **distributions3** package.

Usage

```
CensoredStudentsT(df, location = 0, scale = 1, left = -Inf, right = Inf)
```

Arguments

df	numeric. The degrees of freedom of the underlying <i>uncensored</i> t distribution. Can be any positive number, with df = Inf corresponding to the normal distribution.
location	numeric. The location parameter of the underlying <i>uncensored</i> t distribution, typically written μ in textbooks. Can be any real number, defaults to 0.
scale	numeric. The scale parameter (standard deviation) of the underlying <i>uncensored</i> t distribution, typically written σ in textbooks. Can be any positive number, defaults to 1.
left	numeric. The left censoring point. Can be any real number, defaults to -Inf (uncensored). If set to a finite value, the distribution has a point mass at left whose probability corresponds to the cumulative probability function of the <i>uncensored</i> t distribution at this point.
right	numeric. The right censoring point. Can be any real number, defaults to Inf (uncensored). If set to a finite value, the distribution has a point mass at right whose probability corresponds to 1 minus the cumulative probability function of the <i>uncensored</i> t distribution at this point.

Details

The constructor function `CensoredStudentsT` sets up a distribution object, representing the censored t probability distribution by the corresponding parameters: the degrees of freedom `df`, the latent mean location $= \mu$ and latent scale parameter $\text{scale} = \sigma$ (i.e., the parameters of the underlying *uncensored* t variable), the `left` censoring point (with `-Inf` corresponding to uncensored), and the `right` censoring point (with `Inf` corresponding to uncensored).

The censored t distribution has probability density function (PDF) $f(x)$:

$$\begin{array}{ll} T((left - \mu)/\sigma) & \text{if } x \leq left \\ 1 - T((right - \mu)/\sigma) & \text{if } x \geq right \\ \tau((x - \mu)/\sigma)/\sigma & \text{otherwise} \end{array}$$

where T and τ are the cumulative distribution function and probability density function of the standard t distribution with `df` degrees of freedom, respectively.

All parameters can also be vectors, so that it is possible to define a vector of censored t distributions with potentially different parameters. All parameters need to have the same length or must be scalars (i.e., of length 1) which are then recycled to the length of the other parameters.

For the `CensoredStudentsT` distribution objects there is a wide range of standard methods available to the generics provided in the **distributions3** package: `pdf` and `log_pdf` for the (log-)density (PDF), `cdf` for the probability from the cumulative distribution function (CDF), `quantile` for quantiles, `random` for simulating random variables, `crps` for the continuous ranked probability score (CRPS), and `support` for the support interval (minimum and maximum). Internally, these methods rely on the usual `d/p/q/r` functions provided for the censored t distributions in the **crch** package, see `dct`, and the `crps_ct` function from the **scoringRules** package. The methods `is_discrete` and `is_continuous` can be used to query whether the distributions are discrete on the entire support (always FALSE) or continuous on the entire support (only TRUE if there is no censoring, i.e., if both `left` and `right` are infinite).

See the examples below for an illustration of the workflow for the class and methods.

Value

A `CensoredStudentsT` distribution object.

See Also

[dct](#), [StudentsT](#), [TruncatedStudentsT](#), [CensoredNormal](#), [CensoredLogistic](#)

Examples

```
## package and random seed
library("distributions3")
set.seed(6020)

## three censored t distributions:
## - uncensored standard t with 5 degrees of freedom
## - left-censored at zero with 5 df, latent location = 1 and scale = 1
## - interval-censored in [0, 5] with 5 df, latent location = 2 and scale = 2
X <- CensoredStudentsT(
```

```

    df      = c( 5, 5, 5),
    location = c( 0, 1, 2),
    scale   = c( 1, 1, 2),
    left    = c(-Inf, 0, 0),
    right   = c( Inf, Inf, 5)
  )

X

## compute mean of the censored distribution
mean(X)
## higher moments (variance, skewness, kurtosis) are not implemented yet

## support interval (minimum and maximum)
support(X)

## simulate random variables
random(X, 5)

## histograms of 1,000 simulated observations
x <- random(X, 1000)
hist(x[1, ], main = "uncensored")
hist(x[2, ], main = "left-censored at zero")
hist(x[3, ], main = "interval-censored in [0, 5]")

## probability density function (PDF) and log-density (or log-likelihood)
x <- c(0, 0, 1)
pdf(X, x)
pdf(X, x, log = TRUE)
log_pdf(X, x)

## cumulative distribution function (CDF)
cdf(X, x)

## quantiles
quantile(X, 0.5)

## cdf() and quantile() are inverses (except at censoring points)
cdf(X, quantile(X, 0.5))
quantile(X, cdf(X, 1))

## all methods above can either be applied elementwise or for
## all combinations of X and x, if length(X) = length(x),
## also the result can be assured to be a matrix via drop = FALSE
p <- c(0.05, 0.5, 0.95)
quantile(X, p, elementwise = FALSE)
quantile(X, p, elementwise = TRUE)
quantile(X, p, elementwise = TRUE, drop = FALSE)

## compare theoretical and empirical mean from 1,000 simulated observations
cbind(
  "theoretical" = mean(X),

```

```

    "empirical" = rowMeans(random(X, 1000))
  )

  ## evaluate continuous ranked probability score (CRPS) using scoringRules

  library("scoringRules")
  crps(X, x)

```

clogis

The Censored Logistic Distribution

Description

Density, distribution function, quantile function, and random generation for the left and/or right censored logistic distribution.

Usage

```

dclogis(x, location = 0, scale = 1, left = -Inf, right = Inf, log = FALSE)

pclogis(q, location = 0, scale = 1, left = -Inf, right = Inf,
  lower.tail = TRUE, log.p = FALSE)

qclogis(p, location = 0, scale = 1, left = -Inf, right = Inf,
  lower.tail = TRUE, log.p = FALSE)

rclogis(n, location = 0, scale = 1, left = -Inf, right = Inf)

```

Arguments

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. If length(n) > 1, the length is taken to be the number required.
location	location parameter.
scale	scale parameter.
left	left censoring point.
right	right censoring point.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ otherwise, $P[X > x]$.

Details

If location or scale are not specified they assume the default values of 0 and 1, respectively. `left` and `right` have the defaults `-Inf` and `Inf` respectively.

The censored logistic distribution has density $f(x)$:

$$\begin{aligned} \Lambda((left - \mu)/\sigma) & \quad \text{if } x \leq left \\ 1 - \Lambda((right - \mu)/\sigma) & \quad \text{if } x \geq right \\ \lambda((x - \mu)/\sigma)/\sigma & \quad \text{if } left < x < right \end{aligned}$$

where Λ and λ are the cumulative distribution function and probability density function of the standard logistic distribution respectively, μ is the location of the distribution, and σ the scale.

Value

`dclogis` gives the density, `pclogis` gives the distribution function, `qclogis` gives the quantile function, and `rclogis` generates random deviates.

See Also

[dlogis](#)

cnorm

The Censored Normal Distribution

Description

Density, distribution function, quantile function, and random generation for the left and/or right censored normal distribution.

Usage

```
dcnorm(x, mean = 0, sd = 1, left = -Inf, right = Inf, log = FALSE)
```

```
pcnorm(q, mean = 0, sd = 1, left = -Inf, right = Inf,
  lower.tail = TRUE, log.p = FALSE)
```

```
qcnorm(p, mean = 0, sd = 1, left = -Inf, right = Inf,
  lower.tail = TRUE, log.p = FALSE)
```

```
rcnorm(n, mean = 0, sd = 1, left = -Inf, right = Inf)
```

Arguments

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. If <code>length(n) > 1</code> , the length is taken to be the number required.
<code>mean</code>	vector of means.
<code>sd</code>	vector of standard deviations.
<code>left</code>	left censoring point.
<code>right</code>	right censoring point.
<code>log, log.p</code>	logical; if TRUE, probabilities <code>p</code> are given as <code>log(p)</code> .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ otherwise, $P[X > x]$.

Details

If `mean` or `sd` are not specified they assume the default values of 0 and 1, respectively. `left` and `right` have the defaults `-Inf` and `Inf` respectively.

The censored normal distribution has density $f(x)$:

$$\begin{array}{ll} \Phi((left - \mu)/\sigma) & \text{if } x \leq left \\ 1 - \Phi((right - \mu)/\sigma) & \text{if } x \geq right \\ \phi((x - \mu)/\sigma)/\sigma & \text{if } left < x < right \end{array}$$

where Φ and ϕ are the cumulative distribution function and probability density function of the standard normal distribution respectively, μ is the mean of the distribution, and σ the standard deviation.

Value

`dcnorm` gives the density, `pcnorm` gives the distribution function, `qcnorm` gives the quantile function, and `rcnorm` generates random deviates.

See Also

[dnorm](#)

coef.crch

*Methods for Fitted crch Models***Description**

Methods for extracting information from fitted crch objects.

Usage

```
## S3 method for class 'crch'
coef(object, model = c("full", "location", "scale", "df"), ...)
## S3 method for class 'crch'
vcov(object, model = c("full", "location", "scale", "df"), ...)
## S3 method for class 'crch'
terms(x, model = c("location", "scale", "full"), ...)
## S3 method for class 'crch'
fitted(object, type = c("location", "scale"), ...)
```

Arguments

object, x	an object of class "crch".
model	model for which coefficients shall be returned.
type	type of fitted values.
...	further arguments passed to or from other methods.

Details

In addition to the methods above, a set of standard extractor functions for "crch" objects is available, including methods to the generic functions [print](#), [summary](#), [logLik](#), and [residuals](#).

See Also

[crch](#)

coef.crch.boost

*Methods for Boosted crch Models***Description**

Methods for extracting information from fitted crch.boost objects.

Usage

```
## S3 method for class 'crch.boost'
coef(object, model = c("full", "location", "scale", "df"),
      mstop = NULL, zero.coefficients = FALSE, standardize = FALSE, ...)
## S3 method for class 'crch.boost'
print(x, digits = max(3, getOption("digits") - 3),
      mstop = NULL, zero.coefficients = FALSE, ...)
## S3 method for class 'crch.boost'
summary(object, mstop = NULL, zero.coefficients = FALSE, ...)
## S3 method for class 'crch.boost'
logLik(object, mstop = NULL, ...)
```

Arguments

object, x	an object of class "crch.boost".
model	model for which coefficients shall be returned.
mstop	stopping iteration for which coefficients shall be returned. Can be either a character ("max", "aic", "bic", "cv") or a numeric value.
zero.coefficients	logical whether zero coefficients are returned.
standardize	logical whether coefficients shall be standardized.
digits	the number of significant digits to use when printing.
...	further arguments passed to or from other methods.

Details

In addition to the methods above, the "crch" methods [terms](#), [model.frame](#), [model.matrix](#), [residuals](#), and [fitted](#) can be used also for "crch.boost" objects .

See Also

[crch.boost](#), [coef.crch](#)

Description

Methods for extracting information from fitted hxlr objects.

Usage

```
## S3 method for class 'hxlr'
coef(object, model = c("full", "intercept", "location", "scale"),
      type = c("CLM", "latent"), ...)
## S3 method for class 'hxlr'
vcov(object, model = c("full", "intercept", "location", "scale"),
      type = c("CLM", "latent"), ...)
## S3 method for class 'hxlr'
terms(x, model = c("full", "location", "scale"), ...)
```

Arguments

- object, x an object of class "hxlr".
- model model for which coefficients shall be returned.
- type type of coefficients. Default are CLM type coefficients. For type "latent" coefficients are converted in coefficients for location and scale of the latent distribution (analog to crch models).
- ... further arguments passed to or from other methods.

Details

In addition to the methods above, a set of standard extractor functions for "hxlr" objects is available, including methods to the generic functions [print](#), [summary](#), and [logLik](#).

See Also

[hxlr](#)

crch	<i>Censored and Truncated Regression with Conditional Heteroscedasticity</i>
------	--

Description

Fitting censored (tobit) or truncated regression models with conditional heteroscedasticity.

Usage

```
crch(formula, data, subset, na.action, weights, offset,
      link.scale = c("log", "identity", "quadratic"),
      dist = c("gaussian", "logistic", "student"), df = NULL,
      left = -Inf, right = Inf, truncated = FALSE,
      type = c("ml", "crps"), control = crch.control(...),
      model = TRUE, x = FALSE, y = FALSE, ...)

trch(formula, data, subset, na.action, weights, offset,
```

```

link.scale = c("log", "identity", "quadratic"),
dist = c("gaussian", "logistic", "student"), df = NULL,
left = -Inf, right = Inf, truncated = TRUE,
type = c("ml", "crps"), control = crch.control(...),
model = TRUE, x = FALSE, y = FALSE, ...)

crch.fit(x, z, y, left, right, truncated = FALSE, dist = "gaussian",
  df = NULL, link.scale = "log", type = "ml", weights = NULL, offset = NULL,
  control = crch.control())

```

Arguments

formula	a formula expression of the form $y \sim x \mid z$ where y is the response and x and z are regressor variables for the location and the scale of the fitted distribution respectively.
data	an optional data frame containing the variables occurring in the formulas.
subset	an optional vector specifying a subset of observations to be used for fitting.
na.action	a function which indicates what should happen when the data contain NAs.
weights	optional case weights in fitting.
offset	optional numeric vector with <i>a priori</i> known component to be included in the linear predictor for the location. For <code>crch.fit</code> , <code>offset</code> can also be a list of 2 offsets used for the location and scale respectively.
link.scale	character specification of the link function in the scale model. Currently, "identity", "log", "quadratic" are supported. The default is "log". Alternatively, an object of class "link-glm" can be supplied.
dist	assumed distribution for the dependent variable y .
df	optional degrees of freedom for <code>dist="student"</code> . If omitted the degrees of freedom are estimated.
left	left limit for the censored dependent variable y . If set to <code>-Inf</code> , y is assumed not to be left-censored. Can also be a vector of limits of the same length as y .
right	right limit for the censored dependent variable y . If set to <code>Inf</code> , y is assumed not to be right-censored. Can also be a vector of limits of the same length as y .
truncated	logical. If <code>TRUE</code> truncated model is fitted with <code>left</code> and <code>right</code> interpreted as truncation points, If <code>FALSE</code> censored model is fitted. Default is <code>FALSE</code>
type	loss function to be optimized. Can be either "ml" for maximum likelihood (default) or "crps" for minimum continuous ranked probability score (CRPS).
control	a list of control parameters passed to <code>optim</code> or to the internal boosting algorithm if <code>control=crch.boost()</code> . Default is <code>crch.control()</code> .
model	logical. If <code>TRUE</code> <i>model frame</i> is included as a component of the returned value.
x, y	for <code>crch</code> : logical. If <code>TRUE</code> the model matrix and response vector used for fitting are returned as components of the returned value. for <code>crch.fit</code> : x is a design matrix with regressors for the location and y is a vector of observations.
z	a design matrix with regressors for the scale.
...	arguments to be used to form the default <code>control</code> argument if it is not supplied directly.

Details

crch fits censored (tobit) or truncated regression models with conditional heteroscedasticity with maximum likelihood estimation. Student-t, Gaussian, and logistic distributions can be fitted to left-and/or right censored or truncated responses. Different regressors can be used to model the location and the scale of this distribution. If `control=crch.boost()` optimization is performed by boosting.

trch is a wrapper function for crch with default `truncated = TRUE`.

crch.fit is the lower level function where the actual fitting takes place.

Value

An object of class "crch" or "crch.boost", i.e., a list with the following elements.

coefficients	list of coefficients for location, scale, and df. Scale and df coefficients are in log-scale.
df	if <code>dist = "student"</code> : degrees of freedom of student-t distribution. else NULL.
residuals	the residuals, that is response minus fitted values.
fitted.values	list of fitted location and scale parameters.
dist	assumed distribution for the dependent variable y.
cens	list of censoring points.
optim	output from optimization from optim .
method	optimization method used for optim .
type	used loss function (maximum likelihood or minimum CRPS).
control	list of control parameters passed to optim
start	starting values of coefficients used in the optimization.
weights	case weights used for fitting.
offset	list of offsets for location and scale.
n	number of observations.
nobs	number of observations with non-zero weights.
loglik	log-likelihood.
vcov	covariance matrix.
link	a list with element "scale" containing the link objects for the scale model.
truncated	logical indicating wheter a truncated model has been fitted.
converged	logical variable whether optimization has converged or not.
iterations	number of iterations in optimization.
call	function call.
formula	the formula supplied.
terms	the terms objects used.
levels	list of levels of the factors used in fitting for location and scale respectively.
contrasts	(where relevant) the contrasts used.
y	if requested, the response used.

x if requested, the model matrix used.
 model if requested, the model frame used.
 stepsize, mstop, mstopopt, standardize
 return values of boosting optimization. See [crch.boost](#) for details.

References

Messner JW, Mayr GJ, Zeileis A (2016). Heteroscedastic Censored and Truncated Regression with crch. *The R Journal*, **3**(1), 173–181. doi:[10.32614/RJ2016012](#)

Messner JW, Zeileis A, Broecker J, Mayr GJ (2014). Probabilistic Wind Power Forecasts with an Inverse Power Curve Transformation and Censored Regression. *Wind Energy*, **17**(11), 1753–1766. doi:[10.1002/we.1666](#)

See Also

[predict.crch](#), [crch.control](#), [crch.boost](#)

Examples

```
data("RainIbk", package = "crch")
## mean and standard deviation of square root transformed ensemble forecasts
RainIbk$sqrtensmean <-
  apply(sqrt(RainIbk[,grep('^rainfc',names(RainIbk))]), 1, mean)
RainIbk$sqrtenssd <-
  apply(sqrt(RainIbk[,grep('^rainfc',names(RainIbk))]), 1, sd)

## fit linear regression model with Gaussian distribution
CRCH <- crch(sqrt(rain) ~ sqrtensmean, data = RainIbk, dist = "gaussian")
## same as lm?
all.equal(
  coef(lm(sqrt(rain) ~ sqrtensmean, data = RainIbk)),
  head(coef(CRCH), -1),
  tol = 1e-6)

## print
CRCH
## summary
summary(CRCH)

## left censored regression model with censoring point 0:
CRCH2 <- crch(sqrt(rain) ~ sqrtensmean, data = RainIbk,
  dist = "gaussian", left = 0)

## left censored regression model with censoring point 0 and
## conditional heteroscedasticity:
CRCH3 <- crch(sqrt(rain) ~ sqrtensmean|sqrtenssd, data = RainIbk,
  dist = "gaussian", left = 0)

## left censored regression model with censoring point 0 and
## conditional heteroscedasticity with logistic distribution:
CRCH4 <- crch(sqrt(rain) ~ sqrtensmean|sqrtenssd, data = RainIbk,
```

```

dist = "logistic", left = 0)

## compare AIC
AIC(CRCH, CRCH2, CRCH3, CRCH4)

```

crch.boost

Auxiliary Functions for Boosting crch Models

Description

Auxiliary functions to fit crch models via boosting

Usage

```

crch.boost(maxit = 100, nu = 0.1, start = NULL, dot = "separate",
  mstop = c("max", "aic", "bic", "cv"), nfolds = 10, foldid = NULL,
  maxvar = NULL)

crch.boost.fit(x, z, y, left, right, truncated = FALSE, dist = "gaussian",
  df = NULL, link.scale = "log", type = "ml", weights = NULL, offset = NULL,
  control = crch.boost())

```

Arguments

maxit	the maximum number of boosting iterations.
nu	boosting step size. Default is 0.1.
start	a previously boosted but not converged "crch.boost" object to continue.
dot	character specifying how to process formula parts with a dot (.) on the right-hand side. This can either be "separate" so that each formula part is expanded separately or "sequential" so that the parts are expanded sequentially conditional on all prior parts. Default is "separate"
mstop	method to find optimum stopping iteration. Default is "max" which is maxit. Alternatives are "aic" and "bic" for AIC and BIC optimization and "cv" for cross validation. mstop can also be a positive integer to set the number of boosting iterations. Then maxit is set to mstop and mstop="max".
nfolds	if mstopopt = "cv", number of folds in cross validation.
foldid	if mstopopt = "cv", an optional vector of values between 1 and nfolds identifying the fold each observation is in. If supplied, nfolds can be missing.
maxvar	Positive numeric. Maximum number of parameters to be selected during each iteration (not including intercepts). Used for stability selection.
x, z, y, left, right, truncated, dist, df, link.scale, type, weights, offset, control	see crch.fit for details.

Details

`crch.boost` extends `crch` to fit censored (tobit) or truncated regression models with conditional heteroscedasticity by boosting. If `crch.boost()` is supplied as `control` in `crch` then `crch.boost.fit` is used as lower level fitting function. Note that `crch.control()` with `method=boosting` is equivalent to `crch.boost()`. Thus, boosting can more conveniently be called with `crch(..., method = "boosting")`.

Value

For `crch.boost`: A list with components named as the arguments. For `crch.boost.fit`: An object of class "crch.boost", i.e., a list with the following elements.

<code>coefficients</code>	list of coefficients for location and scale. Scale coefficients are in log-scale. Coefficients are of optimum stopping iteration specified by <code>mstop</code> .
<code>df</code>	if <code>dist = "student"</code> : degrees of freedom of student-t distribution. else <code>NULL</code> .
<code>residuals</code>	the residuals, that is response minus fitted values.
<code>fitted.values</code>	list of fitted location and scale parameters at optimum stopping iteration specified by <code>mstop</code> .
<code>dist</code>	assumed distribution for the dependent variable <code>y</code> .
<code>cens</code>	list of censoring points.
<code>control</code>	list of control parameters.
<code>weights</code>	case weights used for fitting.
<code>offset</code>	list of offsets for location and scale.
<code>n</code>	number of observations.
<code>nobs</code>	number of observations with non-zero weights.
<code>loglik</code>	log-likelihood.
<code>link</code>	a list with element "scale" containing the link objects for the scale model.
<code>truncated</code>	logical indicating wheter a truncated model has been fitted.
<code>iterations</code>	number of boosting iterations.
<code>stepsize</code>	boosting stepsize <code>nu</code> .
<code>mstop</code>	criterion used to find optimum stopping iteration.
<code>mstopopt</code>	optimum stopping iterations for different criteria.
<code>standardize</code>	list of center and scale values used to standardize response and regressors.

References

Messner JW, Mayr GJ, Zeileis A (2017). Non-Homogeneous Boosting for Predictor Selection in Ensemble Post-Processing. *Monthly Weather Review*, **145**(1), 137–147. doi:10.1175/MWRD16-0088.1

See Also

[crch](#), [crch.control](#)

Examples

```
# generate data
suppressWarnings(RNGversion("3.5.0"))
set.seed(5)
x <- matrix(rnorm(1000*20),1000,20)
y <- rnorm(1000, 1 + x[,1] - 1.5 * x[,2], exp(-1 + 0.3*x[,3]))
y <- pmax(0, y)
data <- data.frame(cbind(y, x))

# fit model with maximum likelihood
CRCH <- crch(y ~ .|., data = data, dist = "gaussian", left = 0)

# fit model with boosting
boost <- crch(y ~ .|., data = data, dist = "gaussian", left = 0,
  control = crch.boost(mstop = "aic"))

# more conveniently, the same model can also be fit through
# boost <- crch(y ~ .|., data = data, dist = "gaussian", left = 0,
#   method = "boosting", mstop = "aic")

# AIC comparison
AIC(CRCH, boost)

# summary
summary(boost)

# plot
plot(boost)
```

crch.control

Control Options for crch Models

Description

Auxiliary function for crch fitting. Specifies a list of values passed to [optim](#).

Usage

```
crch.control(method = "BFGS", maxit = NULL, hessian = NULL,
  trace = FALSE, start = NULL, dot = "separate",
  lower = -Inf, upper = Inf, ...)
```

Arguments

method	optimization method passed to optim
maxit	the maximum number of iterations. Default is 5000 except for method="boosting" where the default is 100.

hessian	logical or NULL. If TRUE the numerical Hessian matrix from the <code>optim</code> output is used for estimation of the covariance matrix. If FALSE no covariance matrix is computed. If NULL (the default) the Hessian matrix is computed analytically for <code>dist="gaussian"</code> , <code>dist="logistic"</code> , and <code>dist="student"</code> with predefined <code>df</code> . For <code>dist="student"</code> without prespecified <code>df</code> , no analytical solution is available and a numerical Hessian matrix is forced.
trace	non-negative integer. If positive, tracing information on the progress of the optimization is produced.
start	initial values for the parameters to be optimized over.
dot	character specifying how to process formula parts with a dot (.) on the right-hand side. This can either be "separate" so that each formula part is expanded separately or "sequential" so that the parts are expanded sequentially conditional on all prior parts. Default is "separate"
lower, upper	bounds on the variables for the "L-BFGS-B" method, or bounds in which to search for method "Brent".
...	additional parameters passed to <code>optim</code> .

Value

A list with components named as the arguments.

See Also

`crch`, `optim`

crch.stabsel

Auxiliary Functions for Stability Selection Using Boosting

Description

Auxiliary function which allows to do stability selection on heteroscedastic `crch` models based on `crch.boost`.

Usage

```
crch.stabsel(formula, data, ..., nu = 0.1, q, B = 100, thr = 0.9,
             maxit = 2000, data_percentage = 0.5)
```

Arguments

formula	a formula expression of the form $y \sim x \mid z$ where y is the response and x and z are regressor variables for the location and the scale of the fitted distribution respectively.
data	an optional data frame containing the variables occurring in the formulas.
...	Additional attributes to control the <code>crch</code> model. Note that control is <i>not</i> allowed; <code>crch.stabsel</code> uses <code>crch.boost</code> by default.

nu	Boosting step size (see crch.boost) default is 0.1 as for crch.boost while lower values might yield better results frequently and should be considered.
q	Positive numeric. Maximum number of parameters to be selected during each iteration (not including intercepts).
B	numeric, total number of iterations.
thr	numeric threshold ((0.5-1.0)). Used to generate the new formula and the computation of the per-family error rate.
maxit	Positive numeric value. Maximum number for the boosting algorithm. If q is not reached before maxit the algorithm will stop.
data_percentage	Percentage of data which should be sampled in each of the iterations. Default (and suggested) is 0.5.

Details

[crch.boost](#) allows to perform gradient boosting on heteroscedastic additive models. `crch.stabsel` is a wrapper around the core [crch.boost](#) algorithm to perform stability selection (see references).

Half of the data set (data) is sampled B times to perform boosting (based on [crch.boost](#)). Rather than perform the boosting iterations until a certain stopping criterion is reached (e.g., maximum number of iterations maxit) the algorithm stops as soon as q parameters have been selected. The number of parameters is computed across both parameters location and scale. Intercepts are not counted.

Value

Returns an object of class "stabsel.crch" containing the stability selection summary and the new formula based on the stability selection.

table	A table object containing the parameters which have been selected and the corresponding frequency of selection.
formula.org	Original formula used to perform the stability selection.
formula.new	New formula based including the coefficients selected during stability selection.
family	A list object which contains the distribution-specification from the <code>crch.stabsel</code> call including: dist, cens, and truncated.
parameter	List with the parameters used to perform the stability selection including q, B, thr, p, and PFER (per-family error rate).

References

Meinhausen N, Bühlmann P (2010). Stability selection. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, **72**(4), 417–473. doi:10.1111/j.14679868.2010.00740.x.

See Also

[crch](#), [crch.boost](#)

Examples

```
# generate data
suppressWarnings(RNGversion("3.5.0"))
set.seed(5)
x <- matrix(rnorm(1000*20),1000,20)
y <- rnorm(1000, 1 + x[,1] - 1.5 * x[,2], exp(-1 + 0.3*x[,3]))
y <- pmax(0, y)
data <- data.frame(cbind(y, x))

# fit model with maximum likelihood
CRCH1 <- crch(y ~ .|., data = data, dist = "gaussian", left = 0)

# Perform stability selection
stabsel <- crch.stabsel(y ~ .|., data = data, dist = "gaussian", left = 0,
  q = 8, B = 5)

# Show stability selection summary
print(stabsel); plot(stabsel)

CRCH2 <- crch(stabsel$formula.new, data = data, dist = "gaussian", left = 0 )
BOOST <- crch(stabsel$formula.new, data = data, dist = "gaussian", left = 0,
  control = crch.boost() )

### AIC comparison
sapply( list(CRCH1,CRCH2,BOOST), logLik )
```

Description

Density, distribution function, quantile function, and random generation for the left and/or right censored student-t distribution with df degrees of freedom.

Usage

```
dct(x, location = 0, scale = 1, df, left = -Inf, right = Inf, log = FALSE)

pct(q, location = 0, scale = 1, df, left = -Inf, right = Inf,
  lower.tail = TRUE, log.p = FALSE)

qct(p, location = 0, scale = 1, df, left = -Inf, right = Inf,
  lower.tail = TRUE, log.p = FALSE)

rct(n, location = 0, scale = 1, df, left = -Inf, right = Inf)
```

Arguments

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. If <code>length(n) > 1</code> , the length is taken to be the number required.
<code>location</code>	location parameter.
<code>scale</code>	scale parameter.
<code>df</code>	degrees of freedom (> 0 , maybe non-integer). <code>df = Inf</code> is allowed.
<code>left</code>	left censoring point.
<code>right</code>	right censoring point.
<code>log, log.p</code>	logical; if TRUE, probabilities <code>p</code> are given as <code>log(p)</code> .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ otherwise, $P[X > x]$.

Details

If `location` or `scale` are not specified they assume the default values of 0 and 1, respectively. `left` and `right` have the defaults $-\text{Inf}$ and Inf respectively.

The censored student-t distribution has density $f(x)$:

$$\begin{array}{ll} T((left - \mu)/\sigma) & \text{if } x \leq left \\ 1 - T((right - \mu)/\sigma) & \text{if } x \geq right \\ \tau((x - \mu)/\sigma)/\sigma & \text{if } left < x < right \end{array}$$

where T and τ are the cumulative distribution function and probability density function of the student-t distribution with `df` degrees of freedom respectively, μ is the location of the distribution, and σ the scale.

Value

`dct` gives the density, `pct` gives the distribution function, `qct` gives the quantile function, and `rct` generates random deviates.

See Also

[dt](#)

hxlr

*Heteroscedastic Extended Logistic Regression***Description**

This is a wrapper function for `clm` (from package **ordinal**) to fit (heteroscedastic) extended logistic regression (HXLr) models (Messner et al. 2013).

Usage

```
hxlr(formula, data, subset, na.action, weights, thresholds, link, control, ...)
```

Arguments

formula	a formula expression of the form $y \sim x \mid z$ where y is the response and x and z are regressor variables for the location and the scale of the latent distribution respectively. Response can either be a continuous variable or a factor.
data	an optional data frame containing the variables occurring in the formulas.
subset	an optional vector specifying a subset of observations to be used for fitting.
na.action	a function which indicates what should happen when the data contain NAs. Default is <code>na.omit</code>
weights	optional case weights in fitting.
thresholds	vector of (transformed) thresholds that are used to cut the continuous response into categories. Data frames or matrices with multiple columns are allowed as well. Then each column is used as separate predictor variable for the intercept model.
link	link function, i.e., the type of location-scale distribution assumed for the latent distribution. Default is <code>logit</code> .
control	a list of control parameters passed to <code>optim</code> . Default is <code>hxlr.control</code>
...	arguments to be used to form the default <code>control</code> argument if it is not supplied directly.

Details

Extended logistic regression (Wilks 2009) extends binary logistic regression to multi-category responses by including the thresholds, that are used to cut a continuous variable into categories, in the regression equation. Heteroscedastic extended logistic regression (Messner et al. 2013) extends this model further and allows to add additional predictor variables that are used to predict the scale of the latent logistic distribution.

Value

An object of class "hxr", i.e., a list with the following elements.

coefficients	list of CLM coefficients for intercept, location, and scale model.
fitted.values	list of fitted location and scale parameters.
optim	output from optimization from optim .
method	Optimization method used for optim .
control	list of control parameters passed to optim
start	starting values of coefficients used in the optimization.
weights	case weights used for fitting.
n	number of observations.
nobs	number of observations with non-zero weights.
loglik	log-likelihood.
vcov	covariance matrix.
converged	logical variable whether optimization has converged or not.
iterations	number of iterations in optimization.
call	function call.
scale	the formula supplied.
terms	the terms objects used.
levels	list of levels of the factors used in fitting for location and scale respectively.
thresholds	the thresholds supplied.

References

Messner JW, Mayr GJ, Zeileis A, Wilks DS (2014). Extending Extended Logistic Regression to Effectively Utilize the Ensemble Spread. *Monthly Weather Review*, **142**, 448–456. doi:10.1175/MWRD1300271.1.

Wilks DS (2009). Extending Logistic Regression to Provide Full-Probability-Distribution MOS Forecasts. *Meteorological Applications*, **368**, 361–368.

See Also

[predict.hxr, clm](#)

Examples

```
data("RainIbk", package = "crch")
## mean and standard deviation of square root transformed ensemble forecasts
RainIbk$sqrtensmean <-
  apply(sqrt(RainIbk[,grep('^rainfc',names(RainIbk))]), 1, mean)
RainIbk$sqrtenssd <-
  apply(sqrt(RainIbk[,grep('^rainfc',names(RainIbk))]), 1, sd)

## climatological deciles
```

```

q <- unique(quantile(RainIbk$rain, seq(0.1, 0.9, 0.1)))

## fit ordinary extended logistic regression with ensemble mean as
## predictor variable
XLR <- hxr(sqrt(rain) ~ sqrtensmean, data = RainIbk, thresholds = sqrt(q))
## print
XLR
## summary
summary(XLR)

## fit ordinary extended logistic regression with ensemble mean
## and standard deviation as predictor variables
XLRs <- hxr(sqrt(rain) ~ sqrtensmean + sqrtenssd, data = RainIbk,
  thresholds = sqrt(q))
## fit heteroscedastic extended logistic regression with ensemble
## standard deviation as predictor for the scale
HXR <- hxr(sqrt(rain) ~ sqrtensmean | sqrtenssd, data = RainIbk,
  thresholds = sqrt(q))

## compare AIC of different models
AIC(XLR, XLRs, HXR)

## XLRs and HXR are nested in XLR -> likelihood-ratio-tests
if(require("lmtest")) {
  lrtest(XLR, XLRs)
  lrtest(XLR, HXR)
}

## Not run:
#####
## Cross-validation and bootstrapping RPS for different models
## (like in Messner 2013).
N <- NROW(RainIbk)
## function that returns model fits
fits <- function(data, weights = rep(1, N)) {
  list(
    "XLR" = hxr(sqrt(rain) ~ sqrtensmean, data = data,
      weights = weights, thresholds = sqrt(q)),
    "XLR:S" = hxr(sqrt(rain) ~ sqrtensmean + sqrtenssd, data = data,
      weights = weights, thresholds = sqrt(q)),
    "XLR:SM" = hxr(sqrt(rain) ~ sqrtensmean + I(sqrtensmean*sqrtenssd),
      data = data, weights = weights, thresholds = sqrt(q)),
    "HXR" = hxr(sqrt(rain) ~ sqrtensmean | sqrtenssd, data = data,
      weights = weights, thresholds = sqrt(q)),
    "HXR:S" = hxr(sqrt(rain) ~ sqrtensmean + sqrtenssd | sqrtenssd,
      data = data, weights = weights, thresholds = sqrt(q))
  )
}

## cross validation
id <- sample(1:10, N, replace = TRUE)

```

```

obs <- NULL
pred <- list(NULL)
for(i in 1:10) {
  ## splitting into test and training data set
  trainIndex <- which(id != i)
  testIndex <- which(id == i)
  ## weights that are used for fitting the models
  weights <- as.numeric(table(factor(trainIndex, levels = c(1:N))))
  ## testdata
  testdata <- RainIbk[testIndex,]
  ## observations
  obs <- c(obs, RainIbk$rain[testIndex])
  ## estimation
  modelfits <- fits(RainIbk, weights)
  ## Prediction
  pred2 <- lapply(modelfits, predict, newdata = testdata, type = "cumprob")
  pred <- mapply(rbind, pred, pred2, SIMPLIFY = FALSE)
}
names(pred) <- c(names(modelfits))

## function to compute RPS
rps <- function(pred, obs) {
  OBS <- NULL
  for(i in 1:N)
    OBS <- rbind(OBS, rep(0:1, c(obs[i] - 1, length(q) - obs[i] + 1)))
  apply((OBS-pred)^2, 1, sum)
}

## compute rps
RPS <- lapply(pred, rps, obs = as.numeric(cut(obs, c(-Inf, q, Inf))))

## bootstrapping mean rps
rpsall <- NULL
for(i in 1:250) {
  index <- sample(length(obs), replace = TRUE)
  rpsall <- rbind(rpsall, sapply(RPS, function(x) mean(x[index])))
}

rpssall <- 1 - rpsall/rpsall[,1]
boxplot(rpssall[, -1], ylab = "RPSS", main = "RPSS relative to XLR")
abline(h = 0, lty = 2)

## End(Not run)

```

hxr.control

Control Options for hxr Models

Description

Auxiliary function for hxr fitting. Specifies a list of values passed to [optim](#).

Usage

```
hxlr.control(method = "BFGS", maxit = 5000, hessian = TRUE,
             trace = FALSE, start = NULL, ...)
```

Arguments

method	optimization method used in optim
maxit	the maximum number of iterations.
hessian	logical. Should a numerically differentiated Hessian matrix be returned?
trace	non-negative integer. If positive, tracing information on the progress of the optimization is produced.
start	initial values for the parameters to be optimized over.
...	Additional parameters passed to optim .

Value

A list with components named as the arguments.

See Also

[hxlr](#), [optim](#)

plot.crch.boost	<i>Visualizing Coefficient Paths for Boosted crch Models</i>
-----------------	--

Description

Plot paths of coefficients or log-likelihood contributions for `crch.boost` models.

Usage

```
## S3 method for class 'crch.boost'
plot(x, loglik = FALSE,
     standardize = TRUE, which = c("both", "location", "scale"),
     mstop = NULL, coef.label = TRUE, col = NULL, ...)
```

Arguments

x	an object of class "crch.boost".
loglik	logical whether log-likelihood contribution shall be plotted instead of coefficient value.
standardize	logical whether coefficients shall be standardized. Not used if <code>loglik = TRUE</code>
which	which coefficients: "location" and "scale" plots only the coefficients for the location and scale part of the model respectively. "both" plots the coefficient paths of both parts in one graph.

mstop	Stopping iteration for which a vertical line is plotted. Possible choices are "max", "aic", "bic", "cv", "all", or "no". Default is the stopping iteration used for fitting.
coef.label	logical whether paths shall be labeled.
col	Color(s) for the paths. If which="both" a vector of two colors where the paths for the location are plotted in the first color and for the scale in the second color.
...	further arguments passed to plot.ts .

See Also

[crch.boost](#), [plot.ts](#)

predict.crch	<i>Predictions for Fitted crch Models</i>
--------------	---

Description

Obtains various types of predictions for crch models.

Usage

```
## S3 method for class 'crch'
predict(object, newdata = NULL, type = c("location", "scale",
    "response", "parameter", "density", "probability", "quantile", "crps"),
    na.action = na.pass, at = 0.5, left = NULL, right = NULL, ...)

## S3 method for class 'crch'
prodist(object, newdata = NULL, na.action = na.pass,
    left = NULL, right = NULL, ...)
```

Arguments

object	an object of class "crch".
newdata	an optional data frame in which to look for variables with which to predict.
type	type of prediction: "location" returns the location of the predicted distribution. "scale" returns the scale of the predicted distribution. "response" returns the expected value of the predicted distribution (not equal to location for censored and truncated distributions). "parameter" returns a data frame with predicted location and scale parameters. "density" evaluates the predictive density at at. "probability" evaluates the predictive CDF at at. "quantile" returns a matrix of predicted quantiles with quantile probabilities at. "crps" returns the CRPS of the predictive distributions at at.
na.action	a function which indicates what should happen when the data contain NAs. Default is na.pass

at	a vector of values to evaluate the predictive density (type = "density"), probability (type = "probability"), or CRPS (type = "crps") or a vector of quantile probabilities used for type = "quantile". Alternatively, with at = "function" a function is returned that takes at as an argument.
left	left censoring or truncation point. Only used for type = "quantile". If NULL, censoring or truncation point is obtained from object.
right	right censoring or truncation point. Only used for type = "quantile". If NULL, censoring or truncation point is obtained from object.
...	further arguments passed to or from other methods.

Value

The predict method, for type "response", "location", or "scale", returns a vector with either the location or the scale of the predicted distribution. For type "quantile" a matrix of predicted quantiles each column corresponding to an element of at.

The prodist method returns the fitted/predict probability distribution object.

See Also

[crch](#), [prodist](#)

predict.crch.boost	<i>Predictions for Boosted crch Models</i>
--------------------	--

Description

Obtains various types of predictions for crch.boost models.

Usage

```
## S3 method for class 'crch.boost'
predict(object, newdata = NULL, mstop = NULL, ...)
```

Arguments

object	an object of class "crch.boost".
newdata	an optional data frame in which to look for variables with which to predict.
mstop	stopping iteration. Can be either a character ("max", "aic", "bic", "cv") or a numeric value. If not NULL, newdata has to be supplied.
...	further arguments passed to or from other methods.

Value

For type "response", "location", or "scale" a vector with either the location or the scale of the predicted distribution.

For type "quantile" a matrix of predicted quantiles each column corresponding to an element of at.

See Also

[crch.boost](#), [predict.crch](#)

predict.hxlr

Predictions for Fitted hxlr Models

Description

Obtains various types of predictions/fitted values for heteroscedastic extended logistic regression (HXLr) models.

Usage

```
## S3 method for class 'hxlr'
predict(object, newdata = NULL, type = c("class", "probability",
    "cumprob", "location", "scale"), thresholds = object$thresholds,
    na.action = na.pass, ...)
## S3 method for class 'hxlr'
fitted(object, type = c("class", "probability",
    "cumprob", "location", "scale"), ...)
```

Arguments

object	an object of class "hxlr".
newdata	an optional data frame in which to look for variables with which to predict.
type	type of prediction: "probability" returns a data frame with category probabilities, "cumprob" returns cumulative probabilities, "location" and "scale" return the location and scale of the predicted latent distribution respectively, and "class" returns the category with the highest probability. Default is "class".
thresholds	optional thresholds used for defining the thresholds for types "probability", "cumprob", and "class". Can differ from thresholds used for fitting. If omitted, the same thresholds as for fitting are used.
na.action	A function which indicates what should happen when the data contain NAs. Default is na.pass
...	further arguments passed to or from other methods.

Value

For type "prob" a matrix with number of intervals (= number of thresholds + 1) columns is produced. Each row corresponds to a row in newdata and contains the predicted probabilities to fall in the corresponding interval.

For type "cumprob" a matrix with number of thresholds columns is produced. Each row corresponds to a row in newdata and contains the predicted probabilities to fall below the corresponding threshold.

For types "class", "location", and "scale" a vector is returned respectively with either the most probable categories ("class") or the location ("location") or scale (scale) of the latent distribution.

See Also[hxr](#)

RainIbk

*Precipitation Observations and Forecasts for Innsbruck***Description**

Accumulated 5-8 days precipitation amount for Innsbruck. Data includes GEFS reforecasts (Hamill et al. 2013) and observations from SYNOP station Innsbruck Airport (11120) from 2000-01-01 to 2013-09-17.

Usage

```
data("RainIbk", package = "crch")
```

Format

A data frame with 4977 rows. The first column (rain) are 3 days accumulated precipitation amount observations, Columns 2-12 (rainfc) are 5-8 days accumulated precipitation amount forecasts from the individual ensemble members.

Source

Observations: <https://www.ogimet.com/synops.phtml.en>

Reforecasts: <https://psl.noaa.gov/forecasts/reforecast2/>

References

Hamill TM, Bates GT, Whitaker JS, Murray DR, Fiorino M, Galarneau Jr TJ, Zhu Y, Lapenta W (2013). NOAA's Second-Generation Global Medium-Range Ensemble Reforecast Data Set. *Bulletin of the American Meteorological Society*, **94**(10), 1553-1565.

Examples

```
## Spread skill relationship ##

## load and prepare data
data("RainIbk", package = "crch")

## mean and standard deviation of square root transformed ensemble forecasts
RainIbk$sqrtensmean <-
  apply(sqrt(RainIbk[,grep('^rainfc',names(RainIbk))]), 1, mean)
RainIbk$sqrtenssd <-
  apply(sqrt(RainIbk[,grep('^rainfc',names(RainIbk))]), 1, sd)

## quintiles of sqrtenssd
sdcat <- cut(RainIbk$sqrtenssd, c(-Inf, quantile(RainIbk$sqrtenssd,
```

```

seq(0.2,0.8,0.2)), Inf), labels = c(1:5))

## mean forecast errors for each quintile
m <- NULL
for(i in levels(sdcats)) {
  m <- c(m, mean((sqrt(RainIbk$rain)[sdcats == i] -
    RainIbk$sqrtensmean[sdcats == i])^2, na.rm = TRUE))
}

## plot
boxplot((sqrt(rain) - sqrtensmean)^2~sdcats, RainIbk,
  xlab = "Quintile of ensemble standard deviation",
  ylab = "mean squared error", main = "Spread skill relationship")

```

tlogis

The Truncated Logistic Distribution

Description

Density, distribution function, quantile function, and random generation for the left and/or right truncated logistic distribution.

Usage

```

dtlogis(x, location = 0, scale = 1, left = -Inf, right = Inf, log = FALSE)

ptlogis(q, location = 0, scale = 1, left = -Inf, right = Inf,
  lower.tail = TRUE, log.p = FALSE)

qtlogis(p, location = 0, scale = 1, left = -Inf, right = Inf,
  lower.tail = TRUE, log.p = FALSE)

rtlogis(n, location = 0, scale = 1, left = -Inf, right = Inf)

```

Arguments

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. If length(n) > 1, the length is taken to be the number required.
location	location parameter.
scale	scale parameter.
left	left truncation point.
right	right truncation point.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X ≤ x] otherwise, P[X > x].

Details

If location or scale are not specified they assume the default values of 0 and 1, respectively. `left` and `right` have the defaults `-Inf` and `Inf` respectively.

The truncated logistic distribution has density

$$f(x) = 1/\sigma \lambda((x - \mu)/\sigma) / (\Lambda((right - \mu)/\sigma) - \Lambda((left - \mu)/\sigma))$$

for $left \leq x \leq right$, and 0 otherwise.

Λ and λ are the cumulative distribution function and probability density function of the standard logistic distribution respectively, μ is the location of the distribution, and σ the scale.

Value

`dtlogis` gives the density, `ptlogis` gives the distribution function, `qtlogis` gives the quantile function, and `rtlogis` generates random deviates.

See Also

[dlogis](#)

tnorm	<i>The Truncated Normal Distribution</i>
-------	--

Description

Density, distribution function, quantile function, and random generation for the left and/or right truncated normal distribution.

Usage

```
dtnorm(x, mean = 0, sd = 1, left = -Inf, right = Inf, log = FALSE)
```

```
ptnorm(q, mean = 0, sd = 1, left = -Inf, right = Inf,
       lower.tail = TRUE, log.p = FALSE)
```

```
qtnorm(p, mean = 0, sd = 1, left = -Inf, right = Inf,
       lower.tail = TRUE, log.p = FALSE)
```

```
rtnorm(n, mean = 0, sd = 1, left = -Inf, right = Inf)
```

Arguments

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. If <code>length(n) > 1</code> , the length is taken to be the number required.

<code>mean</code>	vector of means.
<code>sd</code>	vector of standard deviations.
<code>left</code>	left censoring point.
<code>right</code>	right censoring point.
<code>log, log.p</code>	logical; if TRUE, probabilities <code>p</code> are given as $\log(p)$.
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ otherwise, $P[X > x]$.

Details

If `mean` or `sd` are not specified they assume the default values of 0 and 1, respectively. `left` and `right` have the defaults `-Inf` and `Inf` respectively.

The truncated normal distribution has density

$$f(x) = 1/\sigma\phi((x - \mu)/\sigma)/(\Phi((right - \mu)/\sigma) - \Phi((left - \mu)/\sigma))$$

for $left \leq x \leq right$, and 0 otherwise.

Φ and ϕ are the cumulative distribution function and probability density function of the standard normal distribution respectively, μ is the mean of the distribution, and σ the standard deviation.

Value

`dnorm` gives the density, `pnorm` gives the distribution function, `qtnorm` gives the quantile function, and `rtnorm` generates random deviates.

See Also

[dnorm](#)

TruncatedLogistic	Create a Truncated Logistic Distribution
-------------------	--

Description

Class and methods for left-, right-, and interval-truncated logistic distributions using the workflow from the **distributions3** package.

Usage

```
TruncatedLogistic(location = 0, scale = 1, left = -Inf, right = Inf)
```

Arguments

location	numeric. The location parameter of the underlying <i>untruncated</i> logistic distribution, typically written μ in textbooks. Can be any real number, defaults to 0.
scale	numeric. The scale parameter (standard deviation) of the underlying <i>untruncated</i> logistic distribution, typically written σ in textbooks. Can be any positive number, defaults to 1.
left	numeric. The left truncation point. Can be any real number, defaults to -Inf (untruncated). If set to a finite value, the distribution has a point mass at left whose probability corresponds to the cumulative probability function of the <i>untruncated</i> logistic distribution at this point.
right	numeric. The right truncation point. Can be any real number, defaults to Inf (untruncated). If set to a finite value, the distribution has a point mass at right whose probability corresponds to 1 minus the cumulative probability function of the <i>untruncated</i> logistic distribution at this point.

Details

The constructor function `TruncatedLogistic` sets up a distribution object, representing the truncated logistic probability distribution by the corresponding parameters: the latent mean $\text{location} = \mu$ and latent standard deviation $\text{scale} = \sigma$ (i.e., the parameters of the underlying *untruncated* logistic variable), the left truncation point (with -Inf corresponding to untruncated), and the right truncation point (with Inf corresponding to untruncated).

The truncated logistic distribution has probability density function (PDF):

$$f(x) = 1/\sigma \lambda((x - \mu)/\sigma) / (\Lambda((\text{right} - \mu)/\sigma) - \Lambda((\text{left} - \mu)/\sigma))$$

for $\text{left} \leq x \leq \text{right}$, and 0 otherwise, where Λ and λ are the cumulative distribution function and probability density function of the standard logistic distribution, respectively.

All parameters can also be vectors, so that it is possible to define a vector of truncated logistic distributions with potentially different parameters. All parameters need to have the same length or must be scalars (i.e., of length 1) which are then recycled to the length of the other parameters.

For the `TruncatedLogistic` distribution objects there is a wide range of standard methods available to the generics provided in the **distributions3** package: `pdf` and `log_pdf` for the (log-)density (PDF), `cdf` for the probability from the cumulative distribution function (CDF), `quantile` for quantiles, `random` for simulating random variables, `crps` for the continuous ranked probability score (CRPS), and `support` for the support interval (minimum and maximum). Internally, these methods rely on the usual d/p/q/r functions provided for the truncated logistic distributions in the **crch** package, see `dtlogis`, and the `crps_tlogis` function from the **scoringRules** package. The methods `is_discrete` and `is_continuous` can be used to query whether the distributions are discrete on the entire support (always FALSE) or continuous on the entire support (always TRUE).

See the examples below for an illustration of the workflow for the class and methods.

Value

A `TruncatedLogistic` distribution object.

See Also

[dtlogis](#), [Logistic](#), [CensoredLogistic](#), [TruncatedNormal](#), [TruncatedStudentsT](#)

Examples

```
## package and random seed
library("distributions3")
set.seed(6020)

## three truncated logistic distributions:
## - untruncated standard logistic
## - left-truncated at zero with latent location = 1 and scale = 1
## - interval-truncated in [0, 5] with latent location = 2 and scale = 1
X <- TruncatedLogistic(
  location = c( 0, 1, 2),
  scale    = c( 1, 1, 1),
  left     = c(-Inf, 0, 0),
  right    = c( Inf, Inf, 5)
)

X

## compute mean of the truncated distribution
mean(X)
## higher moments (variance, skewness, kurtosis) are not implemented yet

## support interval (minimum and maximum)
support(X)

## simulate random variables
random(X, 5)

## histograms of 1,000 simulated observations
x <- random(X, 1000)
hist(x[1, ], main = "untruncated")
hist(x[2, ], main = "left-truncated at zero")
hist(x[3, ], main = "interval-truncated in [0, 5]")

## probability density function (PDF) and log-density (or log-likelihood)
x <- c(0, 0, 1)
pdf(X, x)
pdf(X, x, log = TRUE)
log_pdf(X, x)

## cumulative distribution function (CDF)
cdf(X, x)

## quantiles
quantile(X, 0.5)

## cdf() and quantile() are inverses (except at truncation points)
```

```

cdf(X, quantile(X, 0.5))
quantile(X, cdf(X, 1))

## all methods above can either be applied elementwise or for
## all combinations of X and x, if length(X) = length(x),
## also the result can be assured to be a matrix via drop = FALSE
p <- c(0.05, 0.5, 0.95)
quantile(X, p, elementwise = FALSE)
quantile(X, p, elementwise = TRUE)
quantile(X, p, elementwise = TRUE, drop = FALSE)

## compare theoretical and empirical mean from 1,000 simulated observations
cbind(
  "theoretical" = mean(X),
  "empirical" = rowMeans(random(X, 1000))
)

## evaluate continuous ranked probability score (CRPS) using scoringRules
library("scoringRules")
crps(X, x)

```

TruncatedNormal

Create a Truncated Normal Distribution

Description

Class and methods for left-, right-, and interval-truncated normal distributions using the workflow from the **distributions3** package.

Usage

```
TruncatedNormal(mu = 0, sigma = 1, left = -Inf, right = Inf)
```

Arguments

mu	numeric. The location parameter of the underlying <i>untruncated</i> normal distribution, typically written μ in textbooks. Can be any real number, defaults to 0.
sigma	numeric. The scale parameter (standard deviation) of the underlying <i>untruncated</i> normal distribution, typically written σ in textbooks. Can be any positive number, defaults to 1.
left	numeric. The left truncation point. Can be any real number, defaults to $-\text{Inf}$ (untruncated). If set to a finite value, the distribution has a point mass at left whose probability corresponds to the cumulative probability function of the <i>untruncated</i> normal distribution at this point.
right	numeric. The right truncation point. Can be any real number, defaults to Inf (untruncated). If set to a finite value, the distribution has a point mass at right whose probability corresponds to 1 minus the cumulative probability function of the <i>untruncated</i> normal distribution at this point.

Details

The constructor function `TruncatedNormal` sets up a distribution object, representing the truncated normal probability distribution by the corresponding parameters: the latent mean $\mu = \mu$ and latent standard deviation $\sigma = \sigma$ (i.e., the parameters of the underlying *untruncated* normal variable), the left truncation point (with `-Inf` corresponding to untruncated), and the right truncation point (with `Inf` corresponding to untruncated).

The truncated normal distribution has probability density function (PDF):

$$f(x) = 1/\sigma\phi((x - \mu)/\sigma)/(\Phi((right - \mu)/\sigma) - \Phi((left - \mu)/\sigma))$$

for $left \leq x \leq right$, and 0 otherwise, where Φ and ϕ are the cumulative distribution function and probability density function of the standard normal distribution respectively.

All parameters can also be vectors, so that it is possible to define a vector of truncated normal distributions with potentially different parameters. All parameters need to have the same length or must be scalars (i.e., of length 1) which are then recycled to the length of the other parameters.

For the `TruncatedNormal` distribution objects there is a wide range of standard methods available to the generics provided in the **distributions3** package: `pdf` and `log_pdf` for the (log-)density (PDF), `cdf` for the probability from the cumulative distribution function (CDF), `quantile` for quantiles, `random` for simulating random variables, `crps` for the continuous ranked probability score (CRPS), and `support` for the support interval (minimum and maximum). Internally, these methods rely on the usual d/p/q/r functions provided for the truncated normal distributions in the **crch** package, see `dtnorm`, and the `crps_tnorm` function from the **scoringRules** package. The methods `is_discrete` and `is_continuous` can be used to query whether the distributions are discrete on the entire support (always FALSE) or continuous on the entire support (always TRUE).

See the examples below for an illustration of the workflow for the class and methods.

Value

A `TruncatedNormal` distribution object.

See Also

`dtnorm`, `Normal`, `CensoredNormal`, `TruncatedLogistic`, `TruncatedStudentsT`

Examples

```
## package and random seed
library("distributions3")
set.seed(6020)

## three truncated normal distributions:
## - untruncated standard normal
## - left-truncated at zero with latent mu = 1 and sigma = 1
## - interval-truncated in [0, 5] with latent mu = 1 and sigma = 2
X <- TruncatedNormal(
  mu    = c( 0, 1, 1),
  sigma = c( 1, 1, 2),
  left  = c(-Inf, 0, 0),
```

```

    right = c( Inf, Inf, 5)
  )

X

## compute mean and variance of the truncated distribution
mean(X)
variance(X)
## higher moments (skewness, kurtosis) are not implemented yet

## support interval (minimum and maximum)
support(X)

## simulate random variables
random(X, 5)

## histograms of 1,000 simulated observations
x <- random(X, 1000)
hist(x[1, ], main = "untruncated")
hist(x[2, ], main = "left-truncated at zero")
hist(x[3, ], main = "interval-truncated in [0, 5]")

## probability density function (PDF) and log-density (or log-likelihood)
x <- c(0, 0, 1)
pdf(X, x)
pdf(X, x, log = TRUE)
log_pdf(X, x)

## cumulative distribution function (CDF)
cdf(X, x)

## quantiles
quantile(X, 0.5)

## cdf() and quantile() are inverses
cdf(X, quantile(X, 0.5))
quantile(X, cdf(X, 1))

## all methods above can either be applied elementwise or for
## all combinations of X and x, if length(X) = length(x),
## also the result can be assured to be a matrix via drop = FALSE
p <- c(0.05, 0.5, 0.95)
quantile(X, p, elementwise = FALSE)
quantile(X, p, elementwise = TRUE)
quantile(X, p, elementwise = TRUE, drop = FALSE)

## compare theoretical and empirical mean from 1,000 simulated observations
cbind(
  "theoretical" = mean(X),
  "empirical" = rowMeans(random(X, 1000))
)

```

```
## evaluate continuous ranked probability score (CRPS) using scoringRules
library("scoringRules")
crps(X, x)
```

TruncatedStudentsT *Create a Truncated Student's T Distribution*

Description

Class and methods for left-, right-, and interval-truncated t distributions using the workflow from the **distributions3** package.

Usage

```
TruncatedStudentsT(df, location = 0, scale = 1, left = -Inf, right = Inf)
```

Arguments

<code>df</code>	numeric. The degrees of freedom of the underlying <i>untruncated</i> t distribution. Can be any positive number, with <code>df = Inf</code> corresponding to the normal distribution.
<code>location</code>	numeric. The location parameter of the underlying <i>untruncated</i> t distribution, typically written μ in textbooks. Can be any real number, defaults to 0.
<code>scale</code>	numeric. The scale parameter (standard deviation) of the underlying <i>untruncated</i> t distribution, typically written σ in textbooks. Can be any positive number, defaults to 1.
<code>left</code>	numeric. The left truncation point. Can be any real number, defaults to <code>-Inf</code> (untruncated). If set to a finite value, the distribution has a point mass at <code>left</code> whose probability corresponds to the cumulative probability function of the <i>untruncated</i> t distribution at this point.
<code>right</code>	numeric. The right truncation point. Can be any real number, defaults to <code>Inf</code> (untruncated). If set to a finite value, the distribution has a point mass at <code>right</code> whose probability corresponds to 1 minus the cumulative probability function of the <i>untruncated</i> t distribution at this point.

Details

The constructor function `TruncatedStudentsT` sets up a distribution object, representing the truncated t probability distribution by the corresponding parameters: the degrees of freedom `df`, the latent mean `location = μ` and latent scale parameter `scale = σ` (i.e., the parameters of the underlying *untruncated* t variable), the `left` truncation point (with `-Inf` corresponding to untruncated), and the `right` truncation point (with `Inf` corresponding to untruncated).

The truncated t distribution has probability density function (PDF) $f(x)$:

$$f(x) = 1/\sigma\tau((x - \mu)/\sigma)/(T((right - \mu)/\sigma) - T((left - \mu)/\sigma))$$

for $left \leq x \leq right$, and 0 otherwise, where T and τ are the cumulative distribution function and probability density function of the standard t distribution with df degrees of freedom, respectively.

All parameters can also be vectors, so that it is possible to define a vector of truncated t distributions with potentially different parameters. All parameters need to have the same length or must be scalars (i.e., of length 1) which are then recycled to the length of the other parameters.

For the TruncatedStudentsT distribution objects there is a wide range of standard methods available to the generics provided in the **distributions3** package: [pdf](#) and [log_pdf](#) for the (log-)density (PDF), [cdf](#) for the probability from the cumulative distribution function (CDF), [quantile](#) for quantiles, [random](#) for simulating random variables, [crps](#) for the continuous ranked probability score (CRPS), and [support](#) for the support interval (minimum and maximum). Internally, these methods rely on the usual d/p/q/r functions provided for the truncated t distributions in the **crch** package, see [dtt](#), and the [crps_tt](#) function from the **scoringRules** package. The methods [is_discrete](#) and [is_continuous](#) can be used to query whether the distributions are discrete on the entire support (always FALSE) or continuous on the entire support (always TRUE).

See the examples below for an illustration of the workflow for the class and methods.

Value

A TruncatedStudentsT distribution object.

See Also

[dtt](#), [StudentsT](#), [CensoredStudentsT](#), [TruncatedNormal](#), [TruncatedLogistic](#)

Examples

```
## package and random seed
library("distributions3")
set.seed(6020)

## three truncated t distributions:
## - untruncated standard t with 5 degrees of freedom
## - left-truncated at zero with 5 df, latent location = 1 and scale = 1
## - interval-truncated in [0, 5] with 5 df, latent location = 2 and scale = 2
X <- TruncatedStudentsT(
  df      = c( 5, 5, 5),
  location = c( 0, 1, 2),
  scale   = c( 1, 1, 2),
  left    = c(-Inf, 0, 0),
  right   = c( Inf, Inf, 5)
)

X

## compute mean of the truncated distribution
mean(X)
## higher moments (variance, skewness, kurtosis) are not implemented yet

## support interval (minimum and maximum)
```

```

support(X)

## simulate random variables
random(X, 5)

## histograms of 1,000 simulated observations
x <- random(X, 1000)
hist(x[1, ], main = "untruncated")
hist(x[2, ], main = "left-truncated at zero")
hist(x[3, ], main = "interval-truncated in [0, 5]")

## probability density function (PDF) and log-density (or log-likelihood)
x <- c(0, 0, 1)
pdf(X, x)
pdf(X, x, log = TRUE)
log_pdf(X, x)

## cumulative distribution function (CDF)
cdf(X, x)

## quantiles
quantile(X, 0.5)

## cdf() and quantile() are inverses (except at truncation points)
cdf(X, quantile(X, 0.5))
quantile(X, cdf(X, 1))

## all methods above can either be applied elementwise or for
## all combinations of X and x, if length(X) = length(x),
## also the result can be assured to be a matrix via drop = FALSE
p <- c(0.05, 0.5, 0.95)
quantile(X, p, elementwise = FALSE)
quantile(X, p, elementwise = TRUE)
quantile(X, p, elementwise = TRUE, drop = FALSE)

## compare theoretical and empirical mean from 1,000 simulated observations
cbind(
  "theoretical" = mean(X),
  "empirical" = rowMeans(random(X, 1000))
)

## evaluate continuous ranked probability score (CRPS) using scoringRules

library("scoringRules")
crps(X, x)

```

Description

Density, distribution function, quantile function, and random generation for the left and/or right truncated student-t distribution with *df* degrees of freedom.

Usage

```
dt(x, location = 0, scale = 1, df, left = -Inf, right = Inf, log = FALSE)
```

```
pt(q, location = 0, scale = 1, df, left = -Inf, right = Inf,
  lower.tail = TRUE, log.p = FALSE)
```

```
qt(p, location = 0, scale = 1, df, left = -Inf, right = Inf,
  lower.tail = TRUE, log.p = FALSE)
```

```
rt(n, location = 0, scale = 1, df, left = -Inf, right = Inf)
```

Arguments

<i>x</i> , <i>q</i>	vector of quantiles.
<i>p</i>	vector of probabilities.
<i>n</i>	number of observations. If <code>length(n) > 1</code> , the length is taken to be the number required.
<i>location</i>	location parameter.
<i>scale</i>	scale parameter.
<i>df</i>	degrees of freedom (> 0 , maybe non-integer). <code>df = Inf</code> is allowed.
<i>left</i>	left censoring point.
<i>right</i>	right censoring point.
<i>log</i> , <i>log.p</i>	logical; if TRUE, probabilities <i>p</i> are given as $\log(p)$.
<i>lower.tail</i>	logical; if TRUE (default), probabilities are $P[X \leq x]$ otherwise, $P[X > x]$.

Details

If *location* or *scale* are not specified they assume the default values of 0 and 1, respectively. *left* and *right* have the defaults `-Inf` and `Inf` respectively.

The truncated student-t distribution has density

$$f(x) = 1/\sigma\tau((x - \mu)/\sigma)/(T((right - \mu)/\sigma) - T((left - \mu)/\sigma))$$

for $left \leq x \leq right$, and 0 otherwise.

where T and τ are the cumulative distribution function and probability density function of the student-t distribution with *df* degrees of freedom respectively, μ is the location of the distribution, and σ the scale.

Value

`dt` gives the density, `pt` gives the distribution function, `qt` gives the quantile function, and `rt` generates random deviates.

See Also[dt](#)

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