

Package ‘cope’

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Description Provides functions to compute and plot Coverage Probability Excursion (CoPE) sets for real valued functions on a 2-dimensional domain. CoPE sets are obtained from repeated noisy observations of the function on the entire domain. They are designed to bound the excursion set of the target function at a given level from above and below with a predefined probability. The target function can be a parameter in spatially-indexed linear regression. Support by NIH grant R01 CA157528 is gratefully acknowledged.

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Contents

ARCoeffMap	2
ComputeCope	3
cope	4

DrawContour	5
ImageMap	6
MBContour	6
plot.cope	7
PlotCope	8
TaylorContour	8
ToyNoise1	9
ToyNoise1Presmooth	10
ToyNoise2	10
ToyNoise3	11
ToyNoiseMap	12
ToySignal	12
ToySignalDC	13
ToySlope	13
Index	14

ARCoeffMap	<i>Generate the AR coefficient map.</i>
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Description

Generate the AR coefficient map.

Usage

ARCoeffMap(Ns = 64)

Arguments

Ns	Number of pixels of the result in one direction. The resulting picture will have Ns x Ns pixels.
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Value

A list containing x and y, the coordinates of the grid and z, a matrix of dimensions Ns x Ns giving the AR coefficients map.

ComputeCope

Compute CoPE sets.

Description

Computes CoPE sets for the data Y using the algorithm from Sommerfeld, Sain and Schwartzman (2015).

Usage

```
ComputeCope(Z, level, X = NULL, w = NULL, correlation = NULL,
  corpar = NULL, groups = NULL, V = NULL, alpha = 0.1, N = 1000,
  mu = NULL, mask = NULL)
```

Arguments

<code>Z</code>	A list with components "x", "y" and "z". Here, x and y are the coordinates of the grid on which the data is observed and z is an array with dimensions $c(\text{length}(x), \text{length}(y), n)$, containing the data. n is the number of observations.
<code>level</code>	The level of interest.
<code>X</code>	The design matrix of the linear model. If NULL, it is set to <code>matrix(rep(1, dim(Y)[3]), ncol=1)</code> corresponding to i.i.d. data.
<code>w</code>	A vector of length <code>nrow(X)</code> indicating the desired linear combination of coefficients to be used in inference, i.e., <code>t(w)</code> NULL, the default is <code>c(1, rep(0, ncol(X) - 1))</code> .
<code>correlation</code>	Type of correlation assumed for the spatially indexed indexed linear models. This is a string that is passed to the function <code>gls</code> from the <code>nlme</code> package. Defaults to NULL which corresponds to i.i.d. errors.
<code>corpar</code>	A list of parameters passed to the correlation function.
<code>groups</code>	A factor vector describing groups that are used in the correlation function. Should have the same length as <code>X</code> .
<code>V</code>	A 4-dimensional array containing the covariance matrix associated with each element of <code>Z\$z</code> . See Details.
<code>alpha</code>	The significance level. Inclusion for CoPE sets holds with probability $1 - \alpha$.
<code>N</code>	The number of bootstrap realizations to generate for determining the threshold.
<code>mu</code>	The true parameter function. Use the default NULL if unknown.
<code>mask</code>	Pixels outside the mask (i.e. where <code>mask == NA</code>) are ignored.

Details

The `V` argument is a 4-dimensional array containing the covariance matrices associated with `Z$z`. Specifically, `V[i, j, , ]` is the covariance matrix of the data in `Z$z[i, j, ]`. If `V` is specified, then the covariance matrix in each element of the array is used to transform `X` and the appropriate element of `Z$z` before fitting the linear model. This is used in place of estimating the covariance matrix withing the `nlme::gls` function.

Value

An object of class `cope`. A list containing the following

- `x`, `y`: The grid coordinates from the input.
- `mu`, `level`, `tau`, `X`, `N`, `alpha`, `mask`: The corresponding values from the input.
- `mu_hat`, `norm_est`: The estimator for `mu` and its normalized version.
- `a_MB`, `a_MB_true`, `a_Tay`, `a_Tay_true`: Thresholds for the CoPE sets determined using the multiplier bootstrap or Taylor's method and the estimated or the true contour, respectively.
- `incl_MB`, `incl_MB_true`, `incl_Tay`, `incl_Tay_true`: Booleans indicating whether inclusion of the excursion set in the upper CoPE set and inclusion of the lower CoPE set in the excursion set holds, when CoPE sets are determined by `a_MB`, `a_MB_true`, `a_Tay` or `a_Tay_true`, respectively. Only available if `mu` is given.

References

M. Sommerfeld, S. Sain and A. Schwartzman. Confidence regions for excursion sets in asymptotically Gaussian random fields, with an application to climate. Preprint, 2015.

Examples

```
# An example using the ToyNoise and ToySignal of this package.
## Not run:
n = 30
Data = ToyNoise1(n = n)
Data$z = Data$z + rep(ToySignal())$z, n)
CopeSet = ComputeCope(Data, level=4/3, mu=ToySignal())$z)
PlotCope(CopeSet)
## End(Not run)
```

`cope`

Coverage Probability Excursion (CoPE) Sets

Description

The package `cope` computes and plots CoPE sets defined in Sommerfeld, Sain and Schwartzman (2015) for 2D functions. CoPE sets for a real-valued target function $\mu(s)$ on a two-dimensional domain are designed to bound the excursion set $\mu(s) \geq c$ from above and below with a predefined probability. The target function can be a parameter in spatially indexed linear regression $Y(s) = X * b(s) + \epsilon(s)$, where s is the spatial location, X is a known n by p design matrix, $\epsilon(s)$ is an error field and $Y(s)$ is the observed data.

Major functions

- [ComputeCope](#) Computes CoPE sets for 2D data.
- [PlotCope](#) Plots CoPE sets.

Toy example functions

- [ToySignal](#) Generates a toy signal.
- [ToyNoise1](#), [ToyNoise2](#), [ToyNoise3](#) Generates realizations of toy noise fields.

References

M. Sommerfeld, S. Sain and A. Schwartzman. Confidence regions for excursion sets in asymptotically Gaussian random fields, with an application to climate. Preprint, 2015.

Examples

```
# An example using the ToySignal and the Toy Noise1 of this package.

# Set sample size.
n = 30
# Generate n realizations of the toy noise field.
Data = ToyNoise1(n = n)
# Add the toy signal to the noise.
Data$z = Data$z + rep(ToySignal()$z, n)
# Compute the CoPE sets.
CopeSet = ComputeCope(Data, level=4/3, mu=ToySignal()$z)
# Plot the result.
PlotCope(CopeSet)
```

DrawContour

Draws the contour $f=c$.

Description

Draws the contour $f=c$.

Usage

```
DrawContour(..., level, col, lty = 1)
```

Arguments

<code>...</code>	An image. Either as a list with components <code>x</code> , <code>y</code> and <code>z</code> or as vectors <code>x</code> and <code>y</code> and a matrix <code>z</code> of dimensions <code>c(length(x), length(y))</code> .
<code>level</code>	The level of the contour.
<code>col</code>	Color of the contour.
<code>lty</code>	Line type for the contour.

ImageMap	<i>Plots an image on a map of the world.</i>
----------	--

Description

Plots an image on a map of the world.

Usage

```
ImageMap(lon, lat, img, mask = NULL, xlab = "Longitude",
         ylab = "Latitude", ...)
```

Arguments

lon	Longitude.
lat	Latitude.
img	The image to plot as a matrix of dimensions <code>c(length(lon),length(lat))</code> .
mask	Matrix of dimensions <code>c(length(lon),length(lat))</code> defining a mask to cut out of the picture.
xlab	Label for the x-axis passed to <code>fields::image.plot</code> . Default is "Longitude".
ylab	Label for the y-axis passed to <code>fields::image.plot</code> . Default is "Latitude".
...	Additional graphical parameters passed to <code>fields::image.plot</code> .

MBContour	<i>Computes Multiplier Bootstrap realizations of the supremum of a Gaussian field on a contour.</i>
-----------	---

Description

Computes Multiplier Bootstrap realizations of the supremum of a Gaussian field on a contour.

Usage

```
MBContour(x, y, R, cont, N = 1000)
```

Arguments

x	x-Coordinates of the grid on which the data is observed.
y	y-Coordinates of the grid on which the data is observed.
R	An array of dimension <code>c(length(x),length(y),n)</code> containing the realizations of the field.
cont	The contours of <code>f</code> at value level
N	The number of Bootstrap realizations to produce. Default is 1000.

Value

A vector of length N containing the Bootstrap realizations of the supremum.

plot.cope	<i>Plots CoPE sets.</i>
-----------	-------------------------

Description

Plots CoPE sets.

Usage

```
## S3 method for class 'cope'
plot(x, ..., taylor = FALSE, use.true.function = FALSE,
     colc = "purple", lwdc = 3, ltyc = 1, colp = "darkred", lwdp = 3,
     ltyp = 1, colm = "darkgreen", lwdm = 3, ltym = 1,
     conlist = list(drawlabels = FALSE))
```

Arguments

x	An object of class cope to be plotted.
...	Additional graphical parameters passed to fields::image.plot.
taylor	Boolean indicating whether the CoPE sets with the threshold obtained by Taylor's method should be plotted. Default is FALSE.
use.true.function	Boolean indicating whether the threshold obtained from the true function should be used. Default is FALSE.
colc	Color of contour line for A_c .
lwdc	Width of contour line for A_c .
ltyc	Type of contour line for A_c .
colp	Color of contour line for $\hat{A}_c^+$.
lwdp	Width of contour line for $\hat{A}_c^+$.
ltyp	Type of contour line for $\hat{A}_c^+$.
colm	Color of contour line for $\hat{A}_c^-$.
lwdm	Width of contour line for $\hat{A}_c^-$.
ltym	Type of contour line for $\hat{A}_c^-$.
conlist	A list of additional arguments to pass to the contour function. By default, the contour labels are not shown.

References

M. Sommerfeld, S. Sain and A. Schwartzman. Confidence regions for excursion sets in asymptotically Gaussian random fields, with an application to climate. Preprint, 2015.

Examples

```
# An example using the ToyNoise and ToySignal of this package.
## Not run:
n = 30
Data = ToyNoise1(n = n)
Data$z = Data$z + rep(ToySignal())$z, n)
CopeSet = ComputeCope(Data, level=4/3, mu=ToySignal())$z)
plot(CopeSet)
## End(Not run)
```

PlotCope	<i>Plots CoPE sets.</i>
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Description

Plots CoPE sets.

Usage

```
PlotCope(cope, plot.taylor = FALSE, use.true.function = FALSE,
         map = FALSE, ...)
```

Arguments

<code>cope</code>	An object of class cope to be plotted.
<code>plot.taylor</code>	Boolean indicating whether the CoPE sets with the threshold obtained by Taylor's method should be plotted. Default is FALSE.
<code>use.true.function</code>	Boolean indicating whether the threshold obtained from the true function should be used. Default is FALSE.
<code>map</code>	If TRUE plot the cope set on a map of the world. The coordinates in this case are interpreted as longitude and latitude.
<code>...</code>	Additional graphical parameters passed to <code>fields::image.plot</code> .

TaylorContour	<i>Computes tail probabilities of a Gaussian field on a contour with Taylor's method.</i>
---------------	---

Description

Computes tail probabilities of a Gaussian field on a contour with Taylor's method.

Usage

```
TaylorContour(x, y, cont, R)
```

Arguments

x	x-Coordinates of the grid on which the data is observed.
y	y-Coordinates of the grid on which the data is observed.
cont	The contour of f at value level
R	An array of dimension c(length(x),length(y),n) containing the realizations of the field.

Value

A function g that computes for $u > 0$ the probability that the supremum of the field exceeds u.

ToyNoise1	<i>Generate a realization of the Toy Noise 1.</i>
-----------	---

Description

Generate a realization of the Toy Noise 1.

Usage

```
ToyNoise1(n = 1, Ns = 64, model = list(), theta = 0.1, l1 = 1,
          l2 = 4, tau = 12)
```

Arguments

n	The number of realizations to produce.
Ns	Number of pixels of the result in one direction. The resulting picture will have Ns x Ns pixels.
model	The correlation structure of the noise, as used by arima.sim. Default is list() which gives i.i.d. noise.
theta	Bandwidth of kernel used to smooth the noise.
l1, l2	Pixel size of the noise blocks in either side of the domain. See main reference for details.
tau	Scaling factor with which noise is multiplied after generation.

Value

A list containing x and y, the coordinates of the grid and z and array of dimensions c(64,64,n) giving n realizations of the Toy Noise 1.

ToyNoise1Presmooth	<i>Generate a realization of the Toy Noise 1 before smoothing.</i>
--------------------	--

Description

Generate a realization of the Toy Noise 1 before smoothing.

Usage

```
ToyNoise1Presmooth(n = 1, Ns = 64, model = list(), theta = 0.1,
  l1 = 1, l2 = 4, tau = 12)
```

Arguments

n	The number of realizations to produce.
Ns	Number of pixels of the result in one direction. The resulting picture will have Ns x Ns pixels.
model	The correlation structure of the noise, as used by arima.sim. Default is list() which gives i.i.d. noise.
theta	Bandwidth of kernel used to smooth the noise.
l1, l2	Pixel size of the noise blocks in either side of the domain. See main reference for details.
tau	Scaling factor with which noise is multiplied after generation.

Value

A list containing x and y, the coordinates of the grid and z and array of dimensions c(64,64,n) giving n realizations of the Toy Noise 1 before smoothing.

ToyNoise2	<i>Generate a realization of the Toy Noise 2.</i>
-----------	---

Description

Generate a realization of the Toy Noise 2.

Usage

```
ToyNoise2(n = 1, Ns = 64, model = list(), theta = 0.1, l1 = 1,
  l2 = 4, tau = 40)
```

Arguments

n	The number of realizations to produce.
Ns	Number of pixels of the result in one direction. The resulting picture will have Ns x Ns pixels.
model	The correlation structure of the noise, as used by arima.sim. Default is list() which gives i.i.d. noise.
theta	Bandwidth of kernel used to smooth the noise.
l1, l2	Pixel size of the noise blocks in either side of the domain. See main reference for details.
tau	Scaling factor with which noise is multiplied after generation.

Value

A list containing x and y, the coordinates of the grid and z and array of dimensions c(64,64,n) giving n realizations of the Toy Noise 2.

ToyNoise3	<i>Generate a realization of the Toy Noise 3.</i>
-----------	---

Description

Generate a realization of the Toy Noise 3.

Usage

```
ToyNoise3(n = 1, Ns = 64, model = list(), theta = 0.05, l1 = 1,
          l2 = 4, tau = 10)
```

Arguments

n	The number of realizations to produce.
Ns	Number of pixels of the result in one direction. The resulting picture will have Ns x Ns pixels.
model	The correlation structure of the noise, as used by arima.sim. Default is list() which gives i.i.d. noise.
theta	Bandwidth of kernel used to smooth the noise.
l1, l2	Pixel size of the noise blocks in either side of the domain. See main reference for details.
tau	Scaling factor with which noise is multiplied after generation.

Value

A list containing x and y, the coordinates of the grid and z and array of dimensions c(64,64,n) giving n realizations of the Toy Noise 3.

ToyNoiseMap	<i>Generate an AR sequence with the ToyFUN as base noise and the AR coefficients given by the ARCoeffMap.</i>
-------------	---

Description

Generate an AR sequence with the ToyFUN as base noise and the AR coefficients given by the ARCoeffMap.

Usage

```
ToyNoiseMap(n = 1, ToyFUN, ...)
```

Arguments

n	Length of sequence.
ToyFUN	Base noise to build the sequence from.
...	Additional parameters passed to ToyFUN.

Value

A list containing x and y, the coordinates of the grid and z and array of dimensions c(64,64,n) giving n realizations of the Toy Noise.

ToySignal	<i>Return the Toy Signal.</i>
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Description

Return the Toy Signal.

Usage

```
ToySignal(ImRange = c(0, 1), NPixel = 64)
```

Arguments

ImRange	A vector with two components giving the range of the region on which the Toy Signal is to be computed.
NPixel	Number of pixels of the result in one direction. The resulting picture will have NPixel x NPixel pixels.

Value

A list with components "x", "y" and "z". Here, x and y are the coordinates of the grid and z is matrix of dimensions c(NPixel,NPixel) giving the Toy Signal.

ToySignalDC	<i>Return the Toy Signal with discontinuities.</i>
-------------	--

Description

Return the Toy Signal with discontinuities.

Usage

```
ToySignalDC(ImRange = c(0, 1), NPixel = 64)
```

Arguments

ImRange	A vector with two components giving the range of the region on which the Toy Signal is to be computed.
NPixel	Number of pixels of the result in one direction. The resulting picture will have NPixel x NPixel pixels.

Value

A list with components "x", "y" and "z". Here, x and y are the coordinates of the grid and z is matrix of dimensions c(NPixel,NPixel) giving the Toy Signal.

ToySlope	<i>The toy slope.</i>
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Description

The toy slope.

Usage

```
ToySlope(ImRange = c(0, 1), NPixel = 64)
```

Arguments

ImRange	A vector with two components giving the range of the region on which the Toy Slope is to be computed.
NPixel	Number of pixels of the result in one direction. The resulting picture will have NPixel x NPixel pixels.

Value

A list with components "x", "y" and "z". Here, x and y are the coordinates of the grid and z is matrix of dimensions c(NPixel,NPixel) giving the Toy Signal.

Index

ARCoeffMap, [2](#)

ComputeCope, [3](#), [4](#)

cope, [4](#)

cope-package (cope), [4](#)

DrawContour, [5](#)

ImageMap, [6](#)

MBContour, [6](#)

plot.cope, [7](#)

PlotCope, [4](#), [8](#)

TaylorContour, [8](#)

ToyNoise1, [5](#), [9](#)

ToyNoise1Presmooth, [10](#)

ToyNoise2, [5](#), [10](#)

ToyNoise3, [5](#), [11](#)

ToyNoiseMap, [12](#)

ToySignal, [5](#), [12](#)

ToySignalDC, [13](#)

ToySlope, [13](#)